Implicit Two-Dimensional Storage Routing Of Floods In A River With Flood Plains

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for the Degree of
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CERTIFICATE

It is certified that the work contained in the thesis entitled "IMPLICIT TWO DIMENSIONAL STORAGE ROUTING OF FLOODS IN A RIVER WITH FLOOD PLAINS" by shri P V N Sudhakar has been carried out under our supervision and this work has not been submitted elsewhere for a degree.

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ABSTRACT

The aim of this thesis is to study the role of flood plains on flood peak subsidence and time of occurrence of peak .A tw dimensional mathematical model of flood routing through a river with flood plains is formulated for this purpose. The model uses only continuity equation and the inertia terms are neglected . The two-dimensionality here does not refer to the unsteady flow equation in two spatial dimensions ,but rather to a physical situation in which river and the flood plains form a two-dimensional network in the horizontal plane . Two laws of exchange that govern the flow between adjacent cells are used in this model depending upon the natural topography . There are river type link and weir type link . River type links are governed by the Manning's formula where as weir type links are governed by the weir discharge formulae . Bed slope is used in the Manning's equation for longitudinal direction where as the water surface slope is used in place of bed slope for transverse direction.The results obtained from the present study are compared with the results obtained by Mahapatra (1990) using a one dimensional model.

The various parameters that are considered in the present study are :

1. ratio of flood plain width to main channel width (B_r).
2. ratio of peak flow to base flow (Q_r).
3. ratio of flood plain roughness to main channel roughness (N_r)

The rate of peak subsidense increases with an increase in N_r value . This effect of Nr is more for higher values of B_r. Rate peak subsidence increases with an increase in Br values . The rate flood peak subsidence decreases with an increase in B_r value Difference in water levels in the transverse direction was found to be small when the water level was much above the flood plain level However significant difference in the water levels of flood plains and main river was found when the flow depth on the flo plains is less than 10 cm .

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LIST OF SYMBOLS

$A_{i,k}$ = Area of flow cross section between cells i and k

A_{si} = Water surface area in the horizontal plane of the i-th cell

ΔA_{si} = Increment of water surface area due to change in water level.

B_r = B_s / B_m

B_m = Width of main channel

B_s = Width of channel including flood plains

cdw = co-efficient of weir

cdo = co-efficient of orifice

dt = Time increment

dx = Length of each cell along longitudinal direction

dz_i = Water level increment

g = Acceleration due to gravity

i,k = Subscripts indicating cells

k = Conveyance of cross section

rn = Mannings n of main channel

fn = Mannings n of flood plains

N_r = fn/rn

P_i = Rainfall intensity

$Q_{i,k}$ = Flow taking place between cells i and k

$R_{i,k}$ = Hydraulic radius of flow cross section across cells i and k

S = Water surface slope

S_o = Longitudinal bed slope

T = Equivelant time

t = Time under consideration

t_{peak} = Time taken to reach peak stage of inflow hydrograph

t_b = Time base of inflow hydrograph

t_g = Time to reach centre of gravity of inflow hydrograph

$\forall_i$ = Volume of water stored in cell i

$\Delta\forall_i$ = Increment of volume of water in cell i

h_b = Height of flood plain above river bed

h_{ir} = Initial uniform flow depth in river

h_{if} = Initial uniform flow depth in flood plains

CHAPTER I

INTRODUCTION

1.1 GENERAL

Floods wreak havoc in many parts of the world and the situation is aggravated by urban and riverine developments .Much of the trauma can be avoided by sound flood plain management ,which calls for sophisticated planning tools . Among the most useful of these are mathematical models . The basic aim of such models is to simulate on a time scale the rise and fall of flood water . Mathematical models can be used to

(i) determine the extent of potential damage ,

(ii) design and operate hydraulic structures properly ,

(iii) predict the peak time ,recession time and peak depths ,and,

(iv) properly manage flood plains .

Mathematical modelling of flood routing is basically the determination of hydrograph (stage or discharge) at any location in the channel if the hydrograph (stage or discharge) is given at an upstream location . The two most important parameters in flood routing are (i) peak depth and (ii) time of occurrence of peak depth. Mathematical modelling of unsteady flow in rivers is much more complicated than in artificial channels because of possible

inundation of flood plains . Flow may not be one - dimensional and it is very difficult to quantify the topographical parameters of inundated areas . Many times it may not be possible to predict the extent of inundation itself , leave alone the other flood characteristics . Straight prismatic channels in which the flow may be considered to be strictly one - dimensional are seldom observed in nature .It is observed that over bank flows go their own way in many cases filling up the flood plains in a manner as dictated by the local topography . In some cases , over bank flow may never return to the main channel during the falling flood .Even if a channel with wel defined flood plains is considered , the flow pattern is not simple . It is seen that the interaction between the main channel and the flood plain flow affects the flood propagation significantly.

Mathematical modelling of flood plain flow should consider the two - dimensional nature of the problem .Two - dimensionality here does not refer to the unsteady flow equations in two spatial dimensions ,but rather to the physical situation in which channel and the storage cell form a two - dimensional network in the horizontal plane .As the flood wave propagates along the main river channel , the water from the main river cells will inundate the flood plain cells .Transfer of water can take place between adjacent flood plain cells also depending on the water levels .During the falling flood the reverse situation occurs i.e. the flow occurs from the flood plain cells towards the main channel,as the river stage lowers first. Most of the earlier models are one - dimensionally designed since two - dimensional models have historically been found to be either

too difficult or too expensive to construct and operate .However the lateral transfer of water due to transverse slopes influences the celerity and consequently the time of occurrence of peak depths (Cunge ,1975) .The flood is also attenuated because of this. Therefore , any accurate model should take into consideration these two - dimensional effects .In the present work ,a two - dimensional mathematical model is developed to study the role of flood plains on flood peak subsidence and time of occurrence of peak depth .

1.2 REVIEW OF LITERATURE

Mathematical modelling of unsteady flow using the complete Saint Venant equations is very common and is covered in most of the text books (Cunge et al 1980 ,Chaudhry 1993) .However, most of these methods have been applied to simple channels , and studies on flood wave movement through compound channels are only few . Verwey (1970) assumed that the overbank portion of flood plains serves only as a storage space and modified the usual one - dimensional Saint Venant equations accordingly . A distinction was made between the " storage width "used in the continuity equation and the " flow width " used in the dynamic equation . Yen (1978) also used a similar concept while studying the subsidence of peak flow in compound channels . He used a diffusion wave model for this purpose . Tingsanchali and Ackermann (1976) considered both the dynamic and storage effects of overbank flow in their one-dimensional model for flood routing in natural rivers.Effect of flood banks was considered through the momemtum correction factor , β and by using Lotter's method for evaluating compound channel resistance . It was observed

that higher peak stages and lower discharges are obtained when only storage effect of flood plains is considered . Later Tingsanchali and Lal (1988) using the same equations obtained semi-empirical exponential equations to describe the subsidence of flood peak discharge in channel during over bank flow periods . Tingsanchali and Ackermann (1976) and Tingsanchali and Lal (1988) used the governing equations in a non-conversation form and the Preissmann scheme for the numerical solution . Mahapatra (1990) and Rao et al. (1992) developed a similar one-dimensional model for flood routing through compound channels . However , they used the governing equations in conservation form which are simpler to integrate and a second - order accurate explicit finite - difference scheme for the numerical solution . Their numerical results indicated that the momemtum correction factor , β does not have significant effect on the flood peak subsidence and it can be taken to one . Mahapatra (1990) also concluded that the effect of inertial terms on flood peak subsidence is negligible and therefore , they can be neglected while routing the floods through compound channels .

One - dimensional models for flood routing through natural channels with flood plains give erroneous results when the flood plains are vast . The interaction between the main channel flow and flood plain flow may result in lateral flow and transverse transfer of water . Mathematical models for flood plain flow should take into account these two - dimensional effects . Zanobetti et al. (1970) constructed and used a two - dimensional model for lower basin of the Mekong river . The model considered a system of ordinary first order

differential equations which represent water level as a function of time in some 300 cells in to which the whole area was divided . Exchange relations between cells were based either on simplified Saint Venant's dynamic equations or on the weir type exchange laws . Inertial terms were ignored in both of these exchange laws because of their insignificant effect . In two - dimensional models of flood plain flow the flood build up is relatively slow (except when the dykes break) and the resistance terms (or friction slope) prevail in the flow equations . It is then logical to drop the inertial terms from tidal equations because they are of small importance as compared to the water surface slope and friction slope . The system of ordinary first order differential equations was solved by an implicit finite - difference method . Later , Cunge et al. (1980) modified this model to take into account the rainfall and the evaporation and applied it to the river Senegal and to the Mono river basin . Hromadka et al. (1985) developed a simple two - dimensional dam-break model for flood plain study purposes . Approximate governing equation based on the diffusion wave approximation were solved using an irregular triangle element integrated finite -difference formulation. The model was applied to simulate a hypothetical dam-break over a flat plain .

Akanbi and Katopodes (1987) developed a two - dimensional model for flood waves propagating on a dry bed . They solve the complete two - dimensional flow equation using a dissipative finite element technique . A deforming grid generation scheme was introduced to account for the effects of propagating or receding fronts . The

developed model was applied to a hypothetical problem involving a flood wave spreading radially on an impervious bed . However , no attempts were made to apply this model for simulating the spreading of flood waters on flood plains . Extension of their model for this purpose and to include the actual topographical information would involve some very elaborate procedures . Also , the computational requirements for the application of such a model are enormous. Leelerc et al. (1990) also presented a finite - element model for estuarian and river flows with moving boundaries . The model defines dry,partly dry and wet type of elements and adopts the governing equations for flow accordingly . The model was tested using a real life application to a tidal flow in the Maniconagan estuary in Canada Recently , Zhao et al. (1994) used a finite-volume method for developing a very versatile two - dimensional unsteady flow model for river basins . In their model , the river basin is discretized using a combination of unstructured triangular and quadrilateral grids . This model can deal with the wetting and drying processes for flood plain and wet land studies . This model was successfully applied to a portion of the Kissimmee river basin in Florida . However , this model requires small computational time steps ,since it uses an explicit scheme for numerical integration . It should be noted here that the models developed by Akanbi and Katapodes (1982) , by Leelerc et al. (1990) and by Zhao et al. (1994) use the complete two - dimensional governing equations and require enormous computational power for their application . This level of sophistication may not be required for many engineering applications where the inertial terms in the governing equations do not have significant effect . In such

cases , the model described by Cunge et al. (1980) is very useful .

1.3 SCOPE OF PRESENT STUDY

Aim of the present study is

(i) to present a two - dimensional mathematical model of a prismatic river with idealized flood plains ,using an implicit finite difference method for the numerical solution .

(ii) to study the role of various channel parameters on the flood wave propagation and the flood peak subsidence and.

(iv) to compare the model with one - dimensional model using inertial effects developed by Mahapatra (1990) .

1.4 CLOSURE

The following chapters present the formulations and use of the two - dimensional mathematical model for flood routing through a river with flood plains . Chapter II presents the governing equations and the numerical methods to solve them . Chapter III describes the applications of the model to a prismatic compound channel and discusses the results . Conclusions and the recommendations for future work are presented in chapter IV .

CHAPTER II

GOVERNING EQUATIONS AND METHOD OF SOLUTION

2.1 INTRODUCTION

In this chapter , the governing equations for two - dimensional storage routing of floods are presented . Numerical scheme for solving these equations is also described .

Presentation in this chapter is organized as follows . In section (2.2) the various assumptions made during the derivation of the governing equations are stated . In section (2.3) the governing equation is derived . In section (2.4) the laws of discharge between adjacent cells are discussed . Numerical scheme for the solution of the governing equations is presented in section (2.5) .

2.2 ASSUMPTIONS

The entire computational domain over which the flood routing computations are to be made is discretized into several inter connected cells . The governing equation for the water level variation in each of the cells is derived based on the following assumptions .

(i) Inertia terms are negligible due to gradual variations in the flow characteristics .

(ii) The volume $\forall_i$ of the water stored in cell is a function of

the height z_i in the cell i.e.

$$\forall_i = \forall (z_i)$$

(iii) The discharge $Q^n_{i,k}$ between two adjacent cells i and k at the given time $t_n = n.\Delta t$ is a function of the levels z^n_i and z^n_k only i.e.

$$Q^n_{i,k} = Q\left(z^n{}_i, z^n{}_k\right)$$

(iv) The discharge between the two cells $Q_{i,k}$ does not vary during the period of time Δt .

(v) The slope , θ of the channel bed is small so that $\sin\theta \approx \tan\theta$ and $\cos\theta \approx 1$.

(vi) The unsteady flow resistance co-efficient is assumed to be the same as for steady flow .

(vii) The velocity distribution is uniform over the flow depth .

(viii) The flow depth is measured at the center and it is uniform throughout the cell .

2.3 DERIVATION OF GOVERNING EQUATION

In this section the continuity equation for a cell is derived . The derivation closely follows the derivation given by Cunge (1976) .

A cell as shown in fig. 2.1 is considered . The water surface elevation in the cell above the datum is z_i . A constant time step Δt is assumed . Then for any given time $t_n = n.\Delta t$ the water level in the cell i is $z_i(t_n)$ and the corresponding water surface area (ABCD) is equal to $A_{si}(t_n)$, the water surface area in the horizontal plane of the cell . The water level is $z_i\left(t_{n+1}\right)$ at the new time level $(n+1).\Delta t$ and the water surface becomes A'B'C'D'. The surface area is equal to $A_{si}\left(t_{n+1}\right) = A_{si}\left(t_n\right) + \Delta A_{si}$. The rainfall $P_i(\tau)$ on

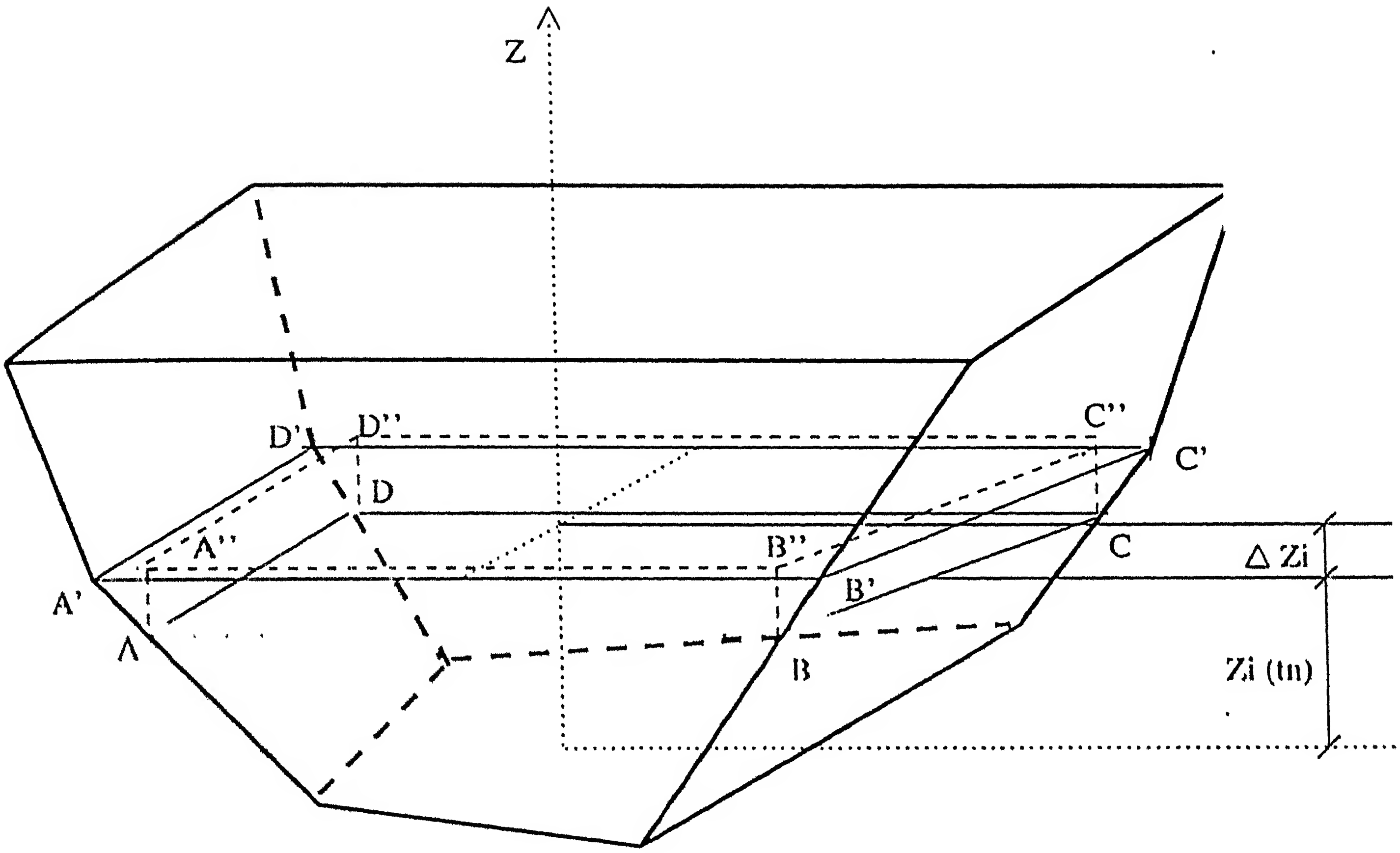


Fig 2.1 Continuity equation of a cell

the cell surface during the time Δt and the discharge $Q_{i,k}\left(z_i, z_k\right)$ from the cells k adjacent to cell i being the reasons behind the increment of the water in the cell . The increase of water volume stored in the cell i during the time Δt may be considered in two ways

from geometrical considerations

$$\Delta \forall_i = \int_{z_i(t_n)}^{z_i(t_n + dt)} A_{si}(z_i) \quad dz_i \qquad (2.1)$$

from discharge considerations

$$\Delta \forall_i = \int_{t_n}^{t_n + dt} P_i \ (t) \ dt \ + \sum_k \int_{t_n}^{t_n + dt} Q_{i,k}(z_i, z_k) \ dt \qquad (2.2)$$

where ($\sum_k$) represents the summation of the volume coming in to cell i from all the cells adjacent to cell i in time Δt . Suppose that the surface A remains unchanged between the two levels z_i and $z_i + \Delta z_i$ i.e. $\left(\frac{\Delta A_{si}}{A_{si}} \ll 1\right)$, then Eq .(2.1) reduces to

$$\Delta \forall_i = A_{si}(z_i) \ \Delta z_i \qquad \text{- - - - - - -} \qquad (2.3)$$

Comparing equation (2.1) and equation (2.2) one can write

$$A_{si}(z_i) \ \Delta z_i = P_i(\tau) \ . \ \Delta t + \Delta t \sum_k Q_{i,k}\left(z_i(\ \tau\ )\ ,\ z_k(\ \tau\ )\right) \qquad (2.4)$$

where , $n.\Delta t \le \tau \le (n+1) \ \Delta t$.

If $\Delta z \longrightarrow 0$, and $\Delta t \longrightarrow 0$, then equation (2.4) can be written in the differential form as -

$$A_{si} \frac{dz_i}{dt} = P_i(t) + \sum_k Q_{i,k} \left(z_i, z_k\right) \qquad (2.5)$$

This is the continuity equation for cell i .Similar equations can be written for all the other cells . The number of equations depends on the number of cells in the computational domain . For example , if

there are N cells then there will be N ordinary equations consisting of N , $(z_1, z_2, - - - - - z_N)$ unknowns .

2.4 LAWS OF DISCHARGE BETWEEN CELLS

Two types of exchanges between cells are generally used .These are (1) River type link and (2) Weir type link. Mathematical representation of these links is described below .

2.4.1 RIVER TYPE LINK

These links represent the exchanges between two cells when there is no local obstacle to the flow and a mean resistance co-efficient for a given cross section of flow can be used . In the case of river type of links , Manning formula as given below is used.

$$Q_{i,k} = \frac{1}{n} \cdot A_{i,k} \cdot R_{i,k}^{2/3} \cdot S^{1/2} \qquad (2.6)$$

where ,

n = Mannings roughness coefficient

$A_{i,k}$ = the area of flow cross section between cells i and k

$R_{i,k}$ = the hydraulic radius of the flow cross section between cells i and k

and S is the water surface slope

In equation (2.6) , parameters $A_{i,k}$ and $R_{i,k}$ are functions of water level $\overline{z}_{i,k}$ in the flow section between cells i and k .This enables one to write -

$$\frac{1}{n} A_{i,k} R_{i,k}^{2/3} = K (\overline{z}_{i,k}) \qquad - - - - - (2.7)$$

where , $\overline{z}_{i,k} = \alpha z_i + (1 - \alpha) z_k$ and the function $K = K(\overline{z}_{i,k}) = K(z_i , z_k)$ is known as the conveyance factor of the flow section

between the cells i and k . α is known as the weighting co-efficient and is constant for a given pair of cells . It is generally calculated such that $\bar{z}_{i,k}$ is the result of a linear interpolation between levels z_i and z_k . In the present study α is taken as 0.5 . The water surface slope in the equation (2.6) is approximated as

$$S = \frac{z_k^{n+1} - z_i^{n+1}}{\Delta x} \quad - - - - - \quad (2.8)$$

where Δx is the fixed center to center distance between the cells i and k .

Now one can write a function $\varnothing$ as

$$\varnothing\ (\bar{z}_{i,k}) = \frac{K\ (\bar{z}_{i,k})}{\sqrt{\Delta x}} = \frac{A\ R^{2/3}}{n\ \sqrt{\Delta x}} \qquad (2.9)$$

The discharge formula , whose sign will depend on the conventions adopted with regard to the direction of flow and may be written as follows -

$$Q_{i,k} = \text{sign}\ (z_k - z_i)\ \varnothing\ (\bar{z}_{i,k})\ \sqrt{\text{abs}(z_k - z_i)} \quad - - - - - (2.10)$$

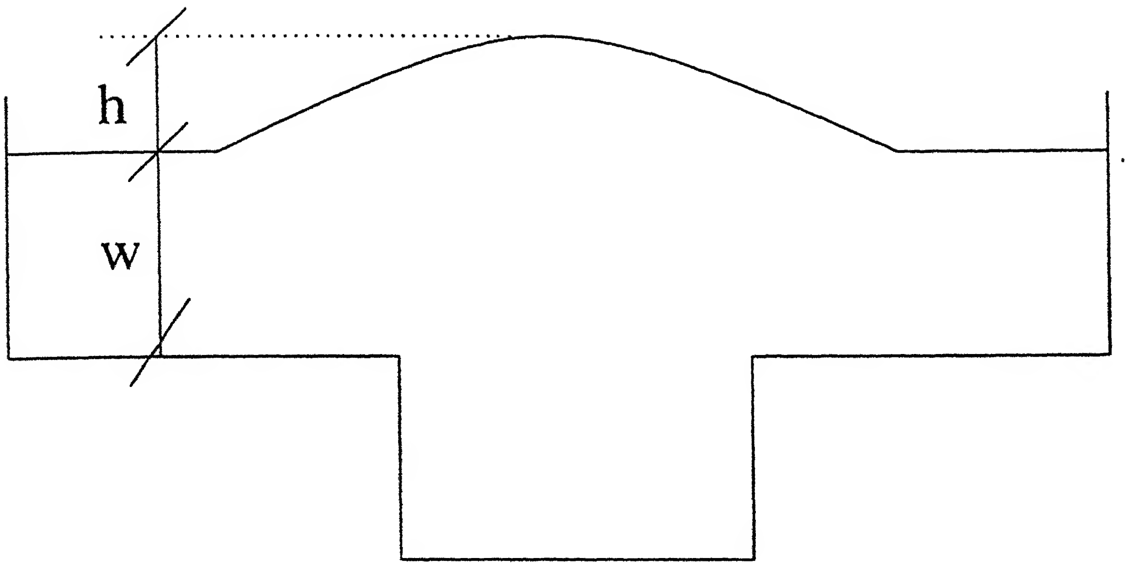
The function $\varnothing$ ($\bar{z}_{i,k}$) must be established a priori based on the channel geometry .

2.4.2 WEIR TYPE LINK

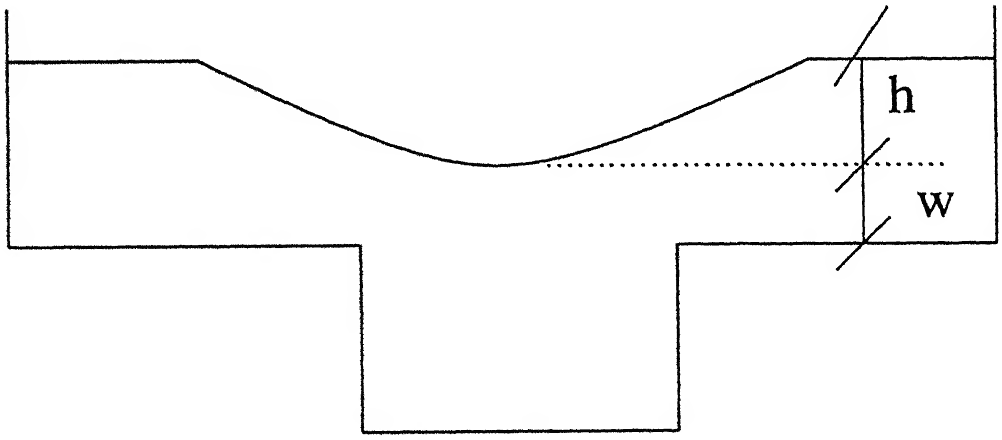
In weir type links one uses the discharge formulae which are applicable to weirs . Two types of flow situations (Figs. 2.2 (i) and 2.2 (ii) may occur for the cases considered in this study . The discharge equations for these are as follows .

(i) if $w < \frac{2}{3}(w+h)$, the flow is considered to be free flow and the discharge equation is given as -

$$Q_{i,k} = c1\ .\ cdw\ .\ h^{3/2} \quad - - - - - (2.11)$$



(i) During rising flood



(ii) During falling flood

Fig 2.2 Weir type link

where , $c1 = \frac{2}{3} \cdot \sqrt{2\,g} \cdot \Delta x$ - - - - - (2.12)

(ii) if $w \geq \frac{2}{3}(w+h)$, the flow is known as submerged flow an the discharge equation is given as -

$$Q_{i,k} = c2 \cdot cdo \cdot w \cdot h^{1/2} \qquad (2.13)$$

where c2 = 1.5 . c1 .In the equations (2.11) and (2.13) cdw and cdo are coefficient of weir and coefficient of orifice and they are given by (Rouse 1936)

$$cdo = 0.611 - 0.175 \left(\frac{w}{h+w}\right) \quad - - - - - (2.14)$$

$$cdw = 0.611 + 0.075 \left(\frac{h}{w}\right) \quad - - - - - \quad (2.15)$$

2.5 NUMERICAL SOLUTION OF GOVERNING EQUATIONS

The governing equations are numerically solved to obtain the water level in any cell as a function of time .An implicit method is adopted in this study so that large values of Δt could be used in the numerical solution . In this method , it is assumed that the discharge $Q_{i,k}\left(z_i(\tau), z_k(\tau)\right)$ is an intermediate discharge between $Q^n_{i,k}$ and $Q^{n+1}_{i,k}$. Now , the water level difference ΔZ_i in Eq.(2.4) cannot be solved explicitly because the right hand side terms contain the values of unknown functions at a time (n+1) Δt . Defining the intermediate value of discharge as

$$Q_{i,k}\left(z_i(\tau), z_k(\tau)\right) = \theta\, Q^{n+1}_{i,k} + (1-\theta)\, Q^n_{i,k} \qquad (2.16)$$

where , $0 \leq \theta \leq 1$

Substitution of equation (2.16) in equation (2.4) leads to

$$\Delta z_i A_{si} = P_i\, \Delta t + \Delta t \left(\theta \sum_k Q^{n+1}_{i,k} + (1-\theta) \sum_k Q^n_{i,k} \right) \qquad (2.17)$$

Discharge equations for the intercell exchange as given by equations

(2.6) , (2.11) and (2.13) are non-linear . Substitution of these equations in the equation (2.17) will result in a system of non-linear equations when written for the cells in the domain . Taylor series expansion can be applied to linearize the equatio assuming that the variation in water level is small during the time interval Δt , making the method of solution easier . Taylor series approximation is applied as shown below

$$F\left(Z_i^{n+1} - Z_k^{n+1}\right) = \left(\frac{Z_i^{n+1} - Z_i^n}{\Delta t}\right) A_{si} - P_i - \theta \sum_k Q_{i,k}^{n+1}$$

$$- - - - \qquad - (1 - \theta) \sum_k Q_{i,k}^n = 0 \quad - - - - - - \quad (2.18)$$

The unknown in Eq.(2.18) are $Z_i^{n+1}$and $Z_k^{n+1}$and Eq.(2.18) can be linearized as

$$\left[\frac{A_{si}}{\Delta t} - \theta \sum_k \left(\frac{\partial Q_{i,k}}{\partial Z_i}\right)\right]_r \Delta Z_i - \left[\theta \sum_k \left(\frac{\partial Q_{i,k}}{\partial Z_k}\right)\right]_r \Delta Z_k$$

$$- - - - - \qquad = - F\left(Z_i, Z_k\right)_r \quad - - - - \quad (2.19)$$

in which the subscript r indicates that the term in the brackets is evaluated at an iteration level r . ΔZ is equal to $Z_{r+1}^{n+1} - Z_r^{n+1}$, which is the improvement in the assumed value of Z at iteration level r.In the equation (2.19) there are N unknown ΔZ values corresponding to N cells , thus making it impossible to solve explicitly . A system of linear algebraic equations for ΔZ_i can be obtained by writing Eq.(2.19) for all cells in the domain . This system of linear algebraic equations is solved simultaneously for each time interval . A considerable amount of computer time would be

necessary if the system of linear equations (2.19) is solved simultaneously by any of the customary methods . However , the problem considered in this study has a special structure which can be exploited to reduce the computational time significantly . The variables Δz_i and Δz_k actually involved in any single equation (2.19) concern only the cell itself and the four adjacent cells .This forms a five point molecule structure having many zero's in the matrix formed by the co - efficients of the variables Δz_i if a systematic numbering of cells is followed . such a system can be efficiently solved by the Strongly Implicit Procedure (Subroutine D03EBF) available in the NAG Library .

The above equations (2.19) are solved simultaneously for Δz values . These values are added to the previous values of Z ,such that the new values are Z = Z + Δz . This procedure is repeated till the Δz values are less than a tolerance specified by us .

CHAPTER - III

TWO - DIMENSIONAL MATHEMATICAL MODEL FOR FLOOD ROUTING THROUGH A RIVER WITH FLOOD PLAINS

3.1 GENERAL

Unsteady flow in a river with flood plains is usually two-dimensional if the flow depth above the flood plains is shallow . In general these flows should be treated as two-dimensional flows for a precise analysis . By two dimensionality , we refer to the physical situation in which the cells form a two dimensional network . However the exchanges of water between the cells are purely one-dimensional . In this chapter , the two-dimensional implicit model is applied to study the role of various parameters of flood plain characterizing the geometry and flow conditions during over bank flow periods . The results obtained from the present study are also compared with the results obtained from one - dimensional model developed by Mahapatra (1990) .

3.2 CHANNEL GEOMETRY

A symmetric prismatic channel with flood plains as shown in Fig. 3.1 is considered for study . This cross section is choosen primarily with a view to compare the results of the two - dimensional model with one - dimensional model studied by Mahapatra (1990) .The depth of the flood plains as measured from the bed of the river , H_1 is taken as 5.0 m . There is no transverse bed slope in the main

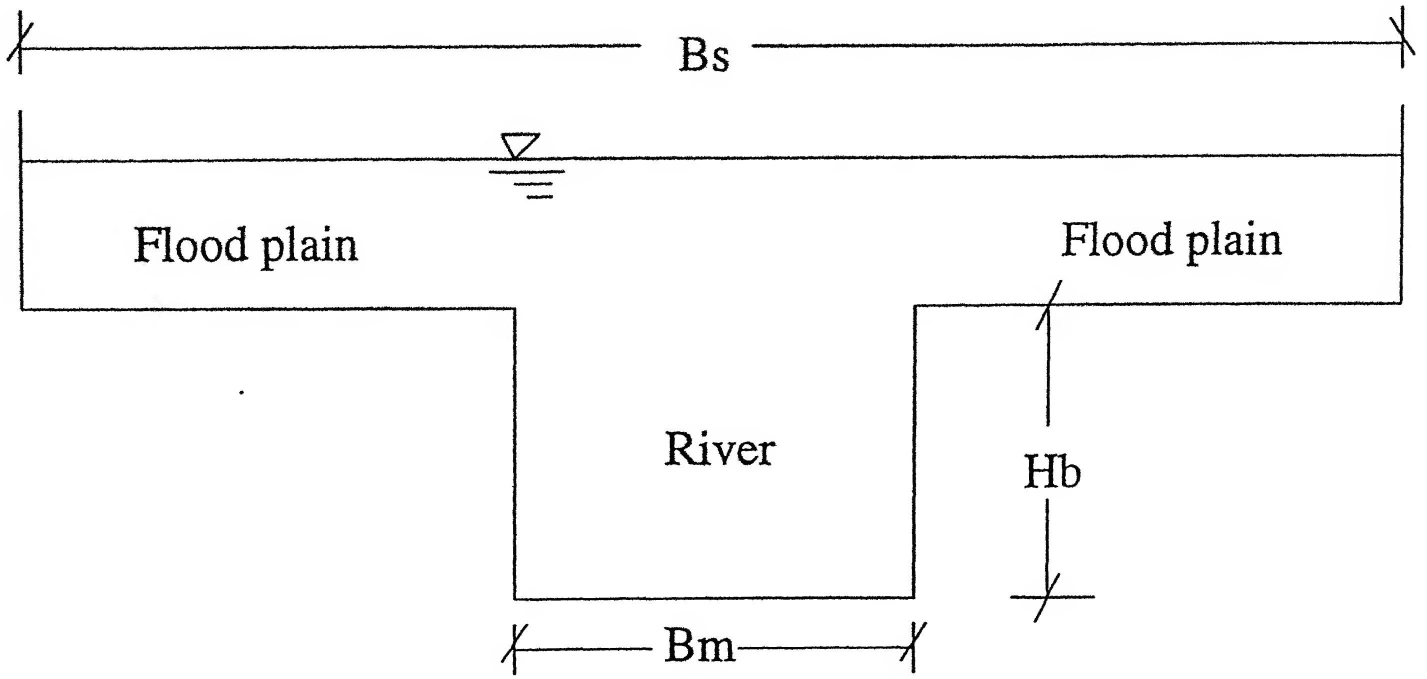


Fig : 3.1 Cross sevtion of River with Flood Plains

channel as well as in the flood plains . However , the longitudinal bed slope remains same for both the flood plains and the main channel The total length of reach is 90 km but results for only first 80 km have been studied in order to prevent the effect of downstream boundary condition . The notations used in figure 3.1 represent

B_m = width of the river

B_s = width of the river including flood plains

H_b = depth of flood plain bed from river bed

3.2.1 IDENTIFICATION OF CELLS

Although the results are represented only for the case of a symmetric prismatic channel , the model is actually developed for any size and shape of the flood plains and the river . The area vectors cross section of each cell and the center to center distance between adjacent cells need to be computed for the application of the two-dimensional model . For this purpose each cell is distinguished by a local node and the local node is in turn represented by its global nodes .In the present case there are 4 global nodes representing each cell placed at their four corners respectively . Once a particular cell i,j is considered then the local node of that cell can be easily prescribed . However , a problem arises with locating the global nodes of that particular cell . This problem is overcome by constructing a connectivity matrix , which connects the local nodes with the respective global nodes . For example , considering the figure (3.2) , there are 6 local nodes and 12 global nodes representing the 6 cells . The connectivity matrix of the figure (3.2) is given as

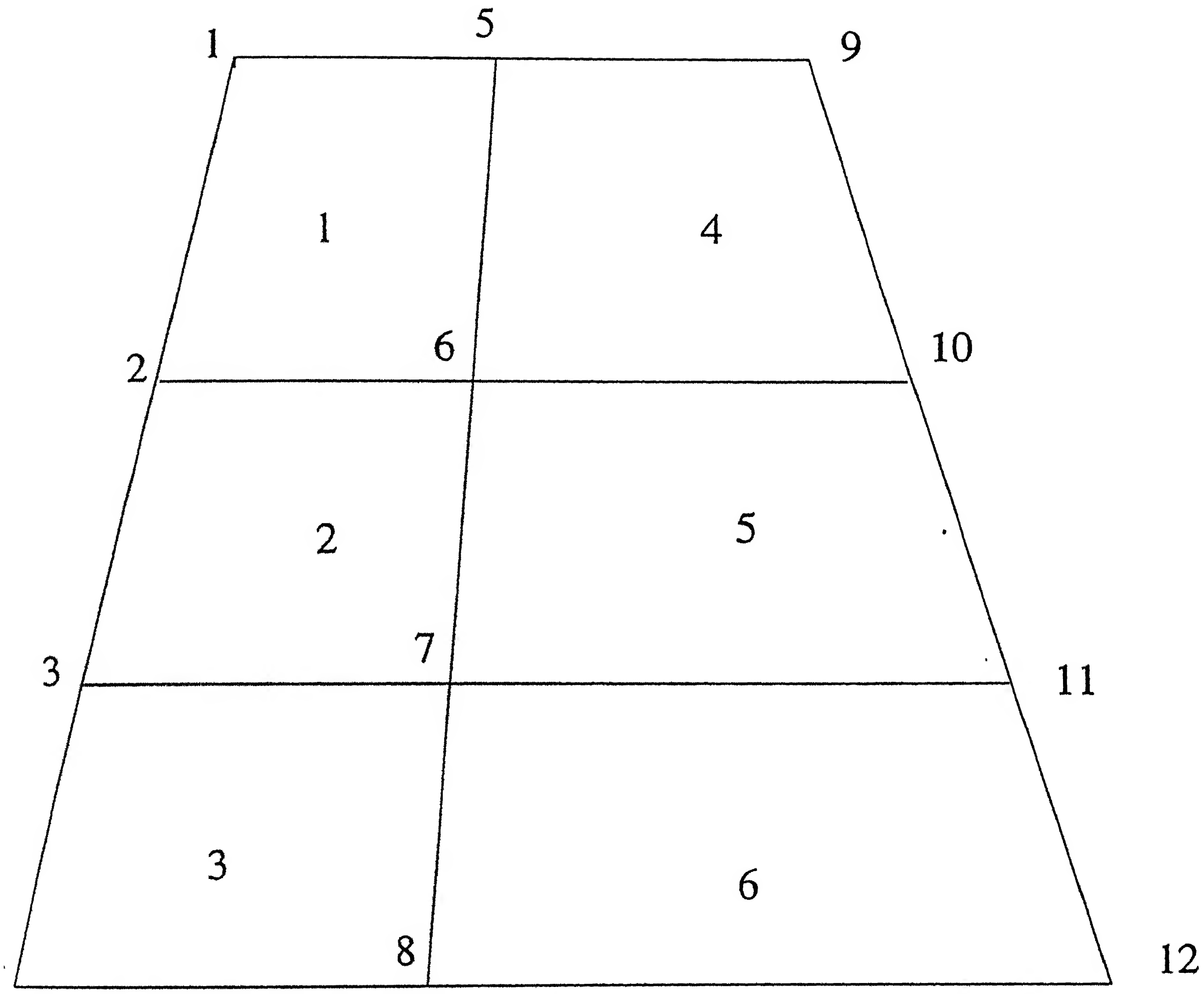


Fig 3.2 Local nodes and Global nodes

$$\begin{bmatrix} 1 & 2 & 6 & 5 \\ 2 & 3 & 7 & 6 \\ 3 & 4 & 8 & 7 \\ 5 & 6 & 10 & 9 \\ 6 & 7 & 11 & 10 \\ 7 & 8 & 12 & 11 \end{bmatrix}$$

The connectivity matrix will contain as many rows as the number of cells present . The number of columns of the connectivity matrix is 4 . The coordinates of these global nodes are also specified as input to facilitate the computation of the area vectors of the intercell faces , plan area of each cell and center to center distance between adjacent cells .

3.3 INITIAL CONDITIONS

The depth of flow at the center of each cell is specified at t=0 as the initial condition . A Steady uniform flow with water level slightly more than the level of the bottom of flood plains is assumed as the initial conditions . The exchanges of flow have been computed based on the river type link or the weir type link depending on the physical topography .River type of links are governed by Mannings equation where as the weir type of links are governed by the weir formula . The channel bottom slope is assumed to be constant along the computational domain and through out the duration of study.

3.4 BOUNDARY CONDITIONS

3.4.1 UPSTREAM

In order to compare the model under study with the one dimensional model developed by Mahapatra (1990) , a hypothetical input hydrograph having a log - pearson type III distribution with four parameters is selected for evaluating the effect of variations in the parameters . The inlow hydrograph is shown in figure (3.3) . The equation of the inflow hydrograph is -

$$Q(t) = Q_b + \left(Q_{peak} - Q_b\right) \left(e^{-(t-t_p)/(t_g-t_p)}\right) \left(\frac{t}{t_p}\right)^{\frac{t_p}{t_g - t_p}}$$

where ,

Q_b= base flow discharge (uniform discharge)

Q_{peak}= peak flow discharge

t_p = time to reach the peak of inflow hydrograph

t_g = time to center of gravity of inflow hydrograph

t = time under consideration

3.4.2 DOWNSTREAM

It is assumed that the change in water level in downstream cells and the cells just preceding them is same . This is equivelant to prescribing a uniform flow condition at the downstream end and is only an approximation of what actually occurs in the field . This approximation procedure is many times resorted to because of non-availability of actual rating curves at the downstream end . The numerical results for the last 10 km of the computational domain are not considered in the analysis because of the above approximate procedure for the downstream boundary condition .

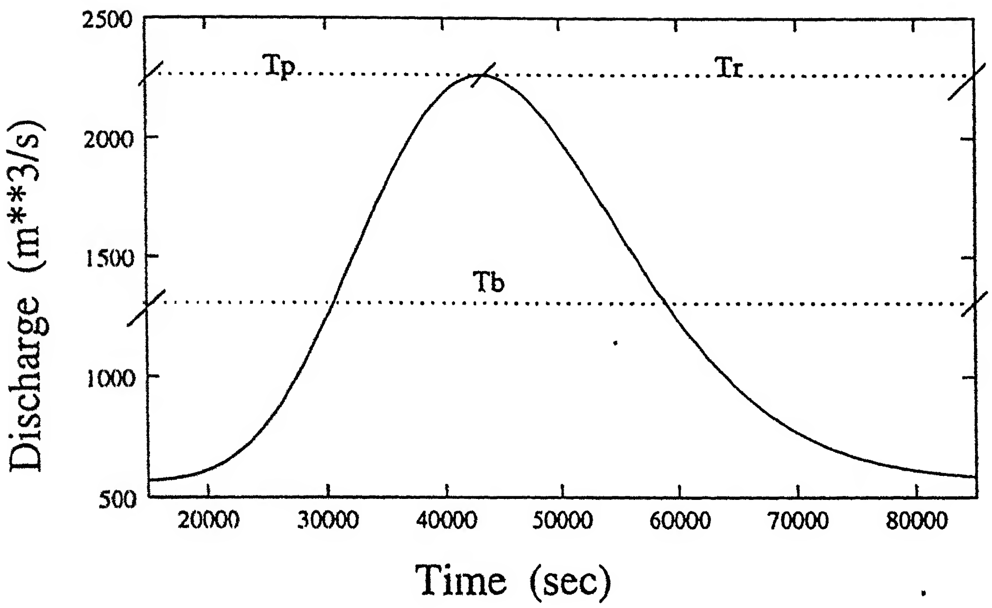


Fig 3.3.1 Inflow hydrograph for Qr =4

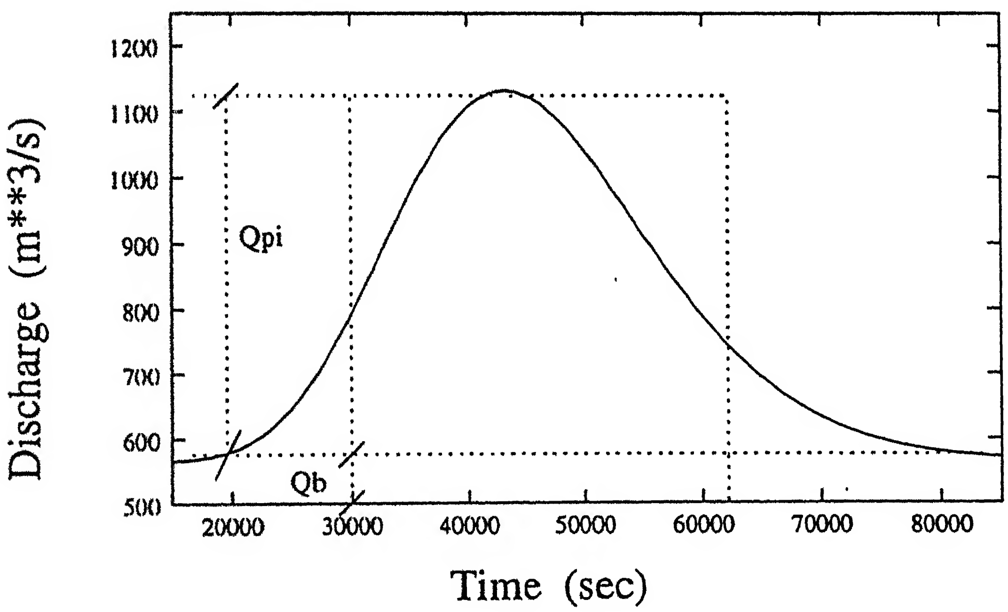


Fig 3.3.2 Inflow hydrograph for Qr = 2

Fig 3.3 Inflow hydrographs

3.5 DEVELOPMENT OF THE MODEL

The total area including the river and the flood plains is divided into number of cells . The task was made simpler by the assumption of well defined flood plains , otherwise the natural topography of the area must be considered while breaking down the given area into cells . For the problem under consideration , there are four types of cells i.e. the river cells , next to river cells , intermediate cells and the side cells (next to side boundaries) . Depending on the geographical topography each cell is linked with the adjacent cells either with a river type of link or with a weir type of link . There is an abrupt change in the bed levels between the river cells and the next to river cells .Therefore,they are connected with weir type of links and the remaining cells are all connected with river type of links . In the present case ,the cell centers are the geometric centres , but depending on the cross section these will change . The direction of flow is allowed from centre to centre of adjacent cells and is determined with the help of area vectors .

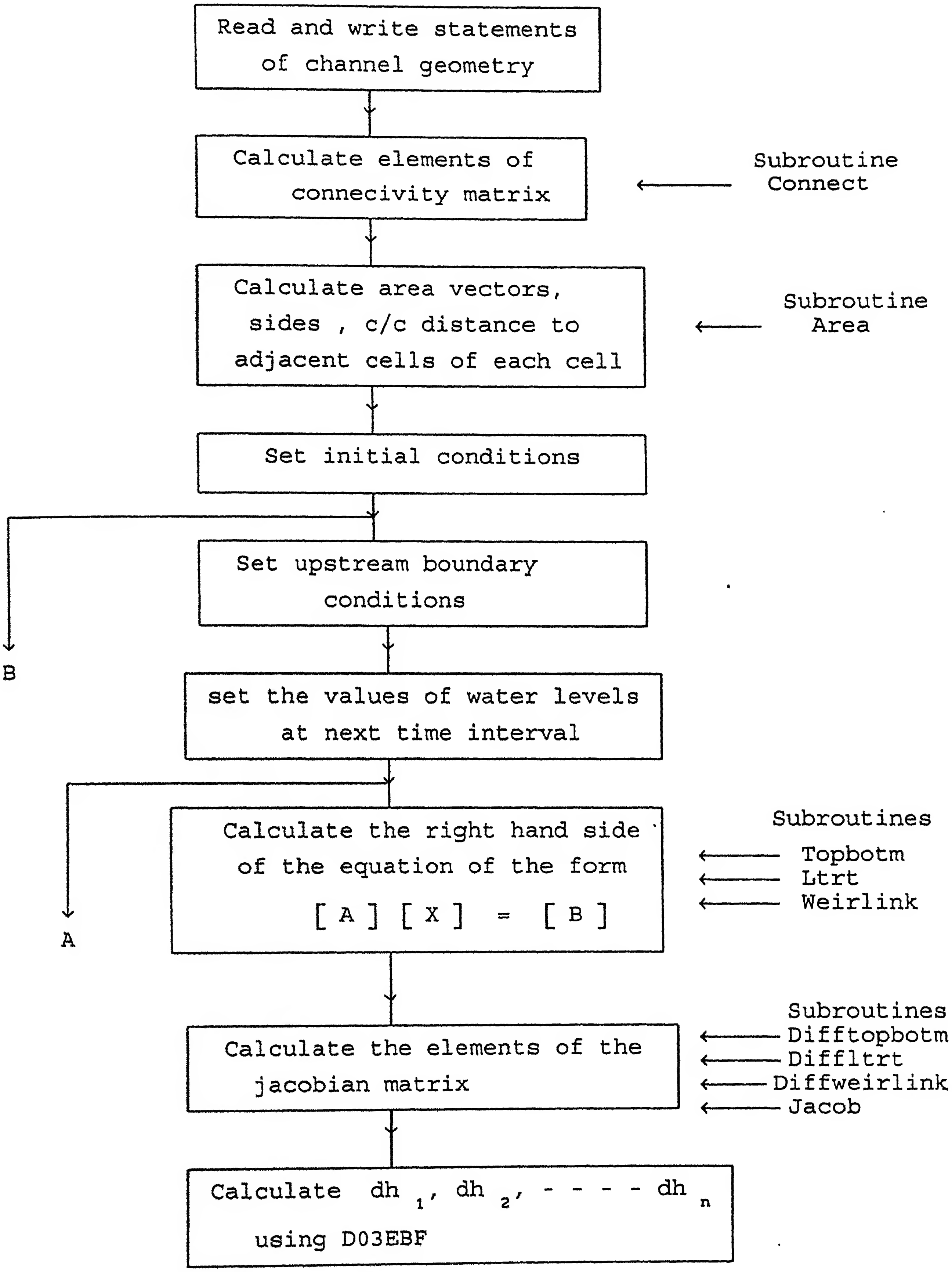
There are five flood plain cells on each side of the river in the transverse direction . The size of the river cells is 4500 m x 50 m . The size of the flood plains in the transverse direction is dependent on the flood plain width considered in a particular run.The Manning's n and the water surface areas are not functions of water but remain constant .The side wall has the same Manning's n as that of flood plains . Bed slope is used along the longitudinal direction where as the water surface slope is used in the transverse direction while considering the river type link .

3.6 COMPUTER PROGRAM

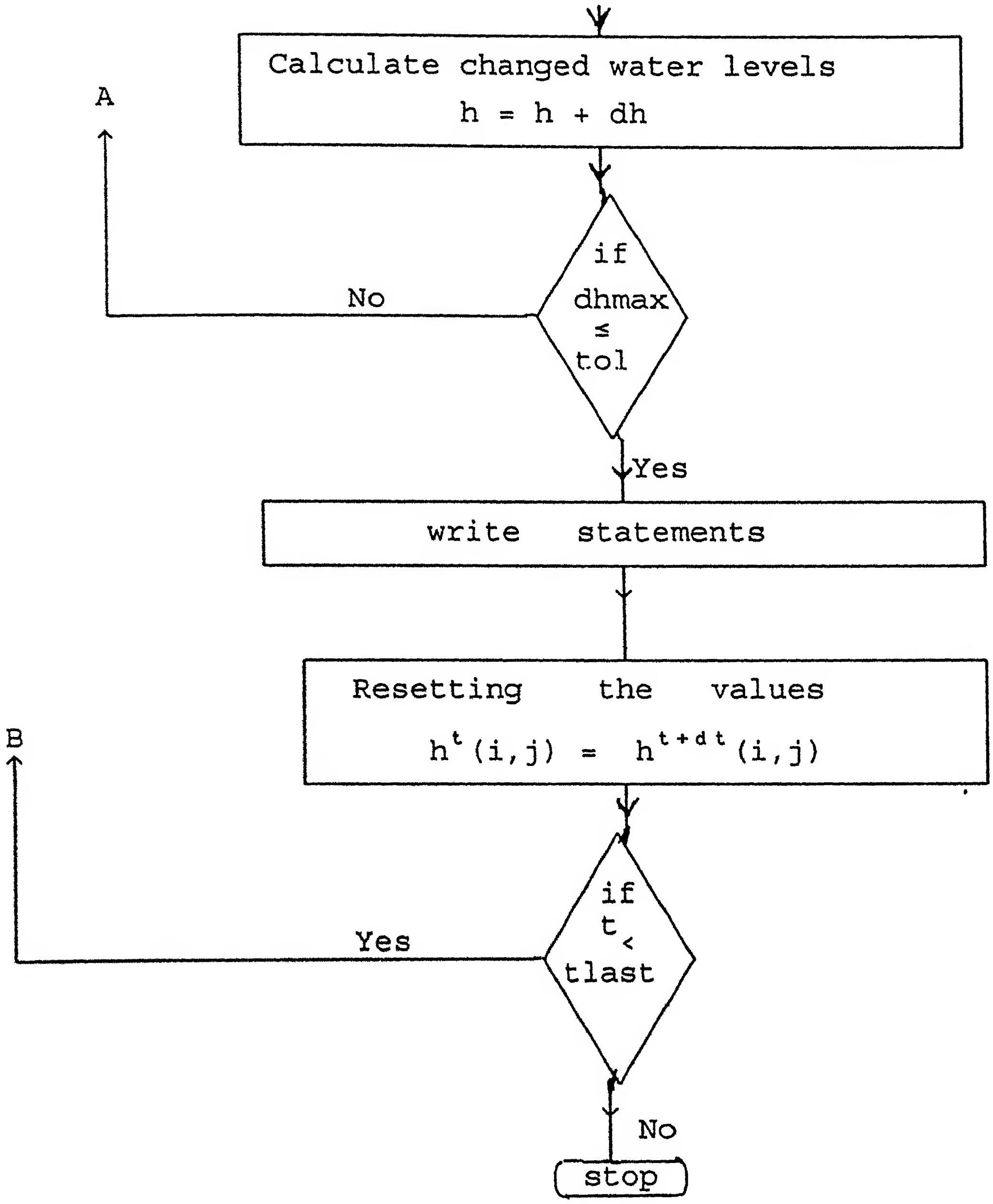
Implicit method is used for solving the governing differential equation for water level variation in a cell . Newton-Raphson iteration method is used for solving the system of non-linear simultaneous algebraic equations . The flow chart is given below and the programme is enclosed in the appendix A .

The main programme calculates the initial conditions , area vectors ,sides of each cell , boundary conditions and solves the set of simultaneous equations by Newtons iterative method at each time step . As mentioned earlier , there are four types of cells depending on their position . They are river cells next to river cells , intermediate cells , and side cells . If one starts writing subroutines based on this classification , they will be lengthy . Any cell is connected to the adjacent cells through one of its four sides ,hence it is simpler to write the subroutines based on this fact . For flow coming in or going out from top and bottom sides of a cell to its adjacent cells a seperate subroutine is written . Another subroutine is written for the flow coming in or going out from left and right sides through river link . Seperate subroutine is written for the weir type of links. Similar subroutines are written for calculating the elements of the jacobian . A seperate subroutine is written for converting these terms in to a matrix form . The elements of the jacobian matrix are later converted into the input form of the D03EBF routine in the main program . The computed stage hydrographs at 0 km ,40.5 km ,76.5 km measured center to center from the upstream cell are considered for the analysis .

FLOW CHART -



Read and write statements of channel geometry
Calculate elements of connecivity matrix
Subroutine Connect
Calculate area vectors, sides , c/c distance to adjacent cells of each cell
Subroutine Area
Set initial conditions
Set upstream boundary conditions
B
set the values of water levels at next time interval
Calculate the right hand side of the equation of the form
[A] [X] = [B]
A
Subroutines
Topbotm
Ltrt
Weirlink
Calculate the elements of the jacobian matrix
Subroutines
Difftopbotm
Diffltrt
Diffweirlink
Jacob
Calculate dh 1, dh 2, - - - - dh n
using D03EBF


Calculate changed water levels
h = h + dh
A
No
if
dhmax
≤
tol
Yes
write statements
Resetting the values
h^t(i,j) = h^{t+dt}(i,j)
B
Yes
if
t <
tlast
No
stop

3.7 SELECTION OF INPUT DATA

A rectangular symmetric , prismatic channel with well defined flood plains as shown in figure 3.1 is considered for the study .The length of the reach is taken as 90 km but the stage hydrographs are drawn up to a distance of 80 km in order to avoid the effect of downstream boundary condition . As shown in the figure 3.1 , the width of the main channel is taken as 50 m and the bank full depth as 5.0 m . The Manning n for the main channel has been taken as 0.03 where as the value of Mannings n for flood plains varied from 0.03 to 0.06 . The longitudinal slope of the main channel and the flood plain are same and equal to 0.0005 . The initial uniform flow depth in the main channel is 5.5 and in the flood plains is taken as 0.5 m .The width of the flood plains is varied from 75 m to 375 m . The datum line is assumed to be horizontal and it is situated at 10 m below the bed of the upstream river cell . The total time of computation is taken as 1,50,000 sec i.e. 42 hrs and the time base of the inflow hydrograph is 70,000sec . The time to travel to peak depth at various downstream locations from the upstream end rate of subsidence depends on the channel parameters and the inflow hydrograph as

$$T_{pl} = f_1 \left(x, h_b, B_m, B_s, rn, fn, g, Q_b, Q_p, t_p, t_g, T \right)$$

$$i = f_2 \left(x, h_b, B_m, B_s, rn, fn, g, Q_b, Q_p, t_p, t_g, T \right)$$

where , i is the subsidence rate ,T_{pl} is the time of travel of the flood peak from the initial station to the various downstream's stations ,x is the distance along the channel ,h_b is the initial uniform flow depth ,B_s is the total width including both the flood plains and river ,$B_m$is the width of the main channel ,rn and fn are

the Manning's roughness coefficients for river and flood plains respectively , g is the acceleration due to gravity , Q_b is the base flow , Q_p is the peak flow , t_p and t_g are the time to reach peak and time to reach centre of gravity of the inflow hydrograph and T is the duration of the equivalent inflow hydrograph . In the above functions T is the time duration of an equivalent rectangle drawn having same volume as that of the inflow hydrograph as shown in Fig.3.3 . It can be written as

$$T = \frac{1}{Q_{pi}} \int_0^{t_b} Q_i\ (t)\ dt$$

For comparison of the results with one dimensional model same formulae are used for normalising the peak depths and peak times . The parameter found for normalising the distance is k_b ,

where $$k_b = \frac{A_i R_i^{2/3}}{B_m\ rn}$$

Various non-dimensional quantities are defined below .

The normalized distance $= \dfrac{x}{\sqrt{k_b T}}$

The flood wave amplitude ratio $= \dfrac{y_{peak)x} - y_{initial}}{y_{peak\ inflow} - y_{initial}}$

normalized time to peak depth is defined as T_{pl} -

$$T_{pl} = \frac{\text{time to peak}_{)x} - \text{time to peak}_{)\text{inflow station}}}{x \Big/ \left(\dfrac{Q_i}{A_i} + \sqrt{g\ hir} \right)}$$

where Q_i and A_i are calculated from the initial uniform depth in the river i.e. hir .

The variation of parameters studied in this case are -

$$n_r = \frac{fn}{rn} , \qquad Q_r = \frac{Q_p}{Q_b} , \qquad B_r = \frac{B_s}{B_m}$$

These parameters are varied one by one and other parameters are kept constant . The results obtained in this study were compared with those obtained using one dimensional model .

3.8 COMPUTATION

The total channel reach is divided in to 20 cells in the longitudinal direction i.e. dx = 4500 km and in to 11 cells in the transverse direction.The value of time step Δt was taken as 1000 sec. There were altogether eight sets of parameters studied in the present case , they are as follows -

sl. no.	$Qr = \frac{Q_p}{Q_b}$	$Br = \frac{B_s}{B_m}$	$Nr = \frac{fn}{rn}$
1	4	4	1
2	4	4	2
3	4	16	1
4	4	16	2
5	2	4	1
6	2	4	2
7	2	16	1
8	2	16	2

In all the above cases ,the values of the following parameters remain unchanged unless and otherwise specified.

B_m = 50 m $\qquad$ g = 9.81 m/s^2

h_b = 5.0 m $\qquad$ hir = 5.5 m

Δx = 4500 m hif = 0.5 m

S_o = 0.0005 t_g = 46,000 sec

Q_b = 565.5668 m^3/s t_p = 43,200 sec

rn = 0.03 tlast = 1,50,000 sec

Q_i = 565.5668 m^3/s

3.9 RESULTS AND DISCUSSION

Stage hydrographs are plotted at 0 km , 40.5 km and 76.5 km from upstream end in figures 3.4 - 3.11 for different parameter combinations . Fig. 3.12 shows the water surface profiles along the river at different times.These plots reveal that peak depth decreases and time to reach peak depth increases along the river . This qualitatively validates the two-dimensional model developed in the present study . The results of the two - dimensional model were compared with that of one - dimensional model .

The results of the numerical simulations to study the effect of Q_r , B_r and N_r on the peak depth are shown in the figures 3.13 - 3.20 . The normalized distance is plotted on x axis and the flood wave amplitude ratio is plotted on y-axis in these figures . In figures 3.13 - 3.14 the effect of B_r is seen on the flood peak depth . The rate of flood peak subsidence for B_r is observed to be more than that for B_r = 4 due to the availability of more storage in the flood plains . Similar trend is observed in the one dimensional model . In figures 3.15 - 3.17 the effect of N_r is observed . It is observed in all the three figures that higher the value of N_r higher is the rate of peak subsidence . Similar result

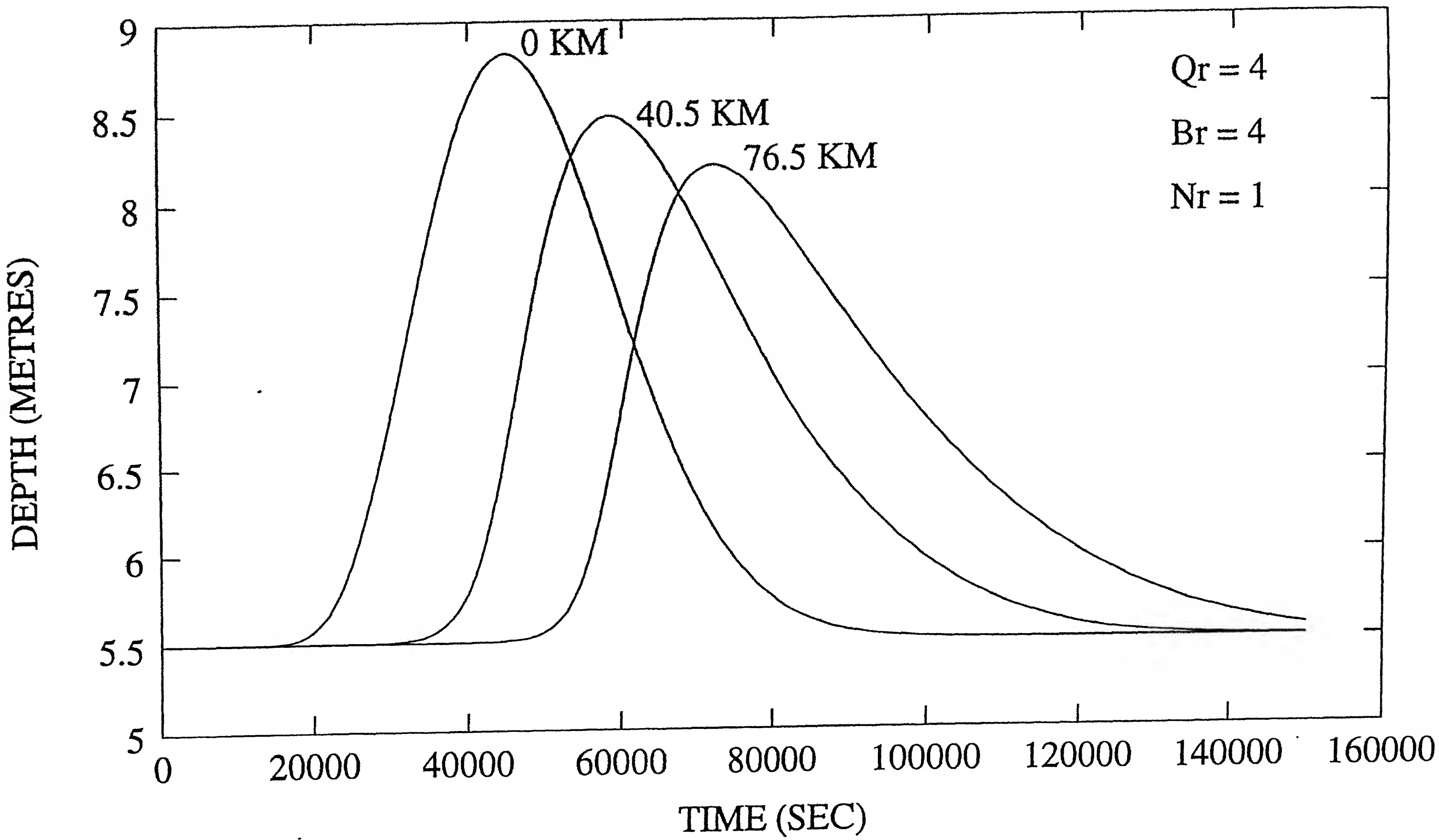


Fig 3.4 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

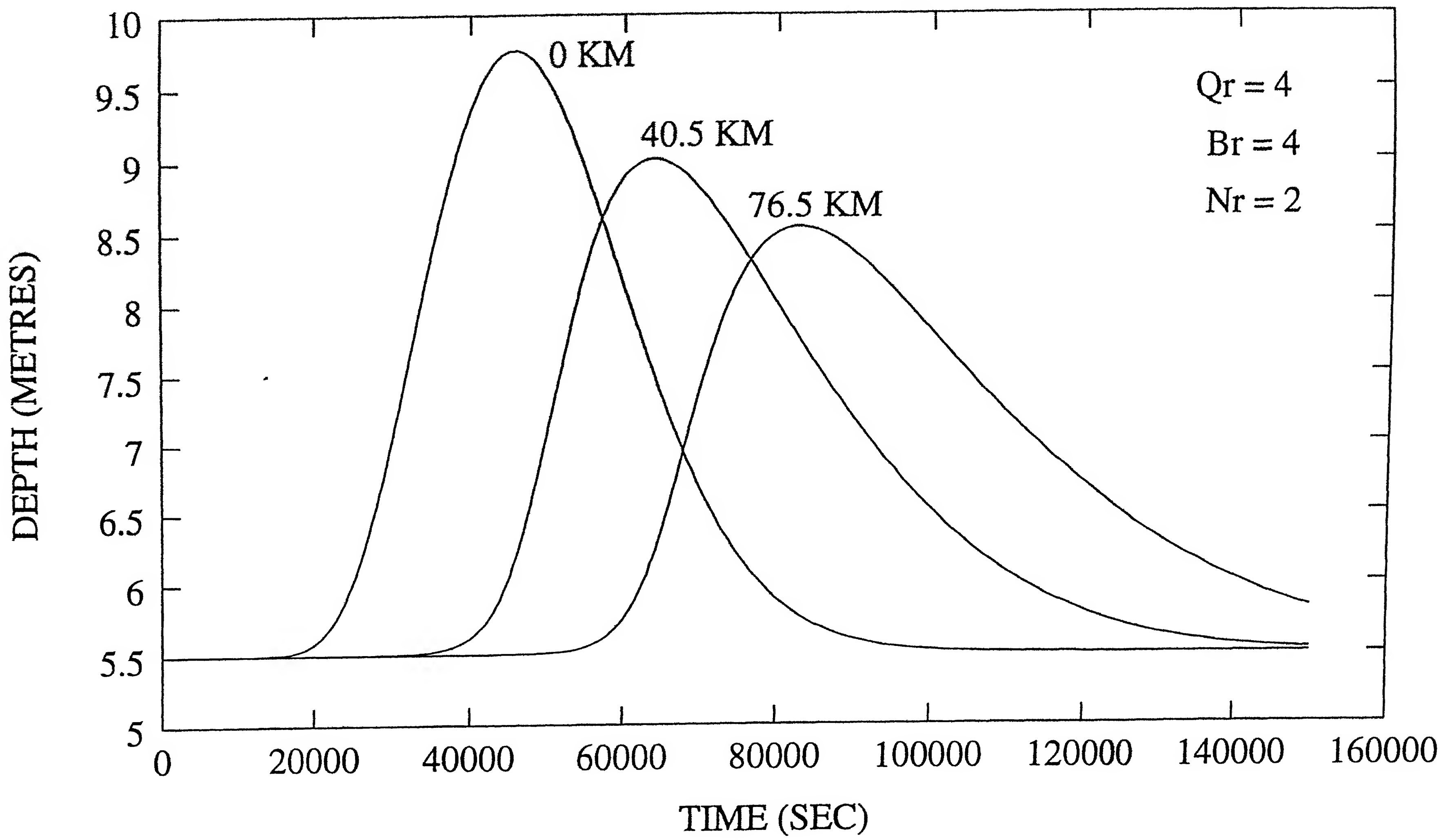


Fig 3.5 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

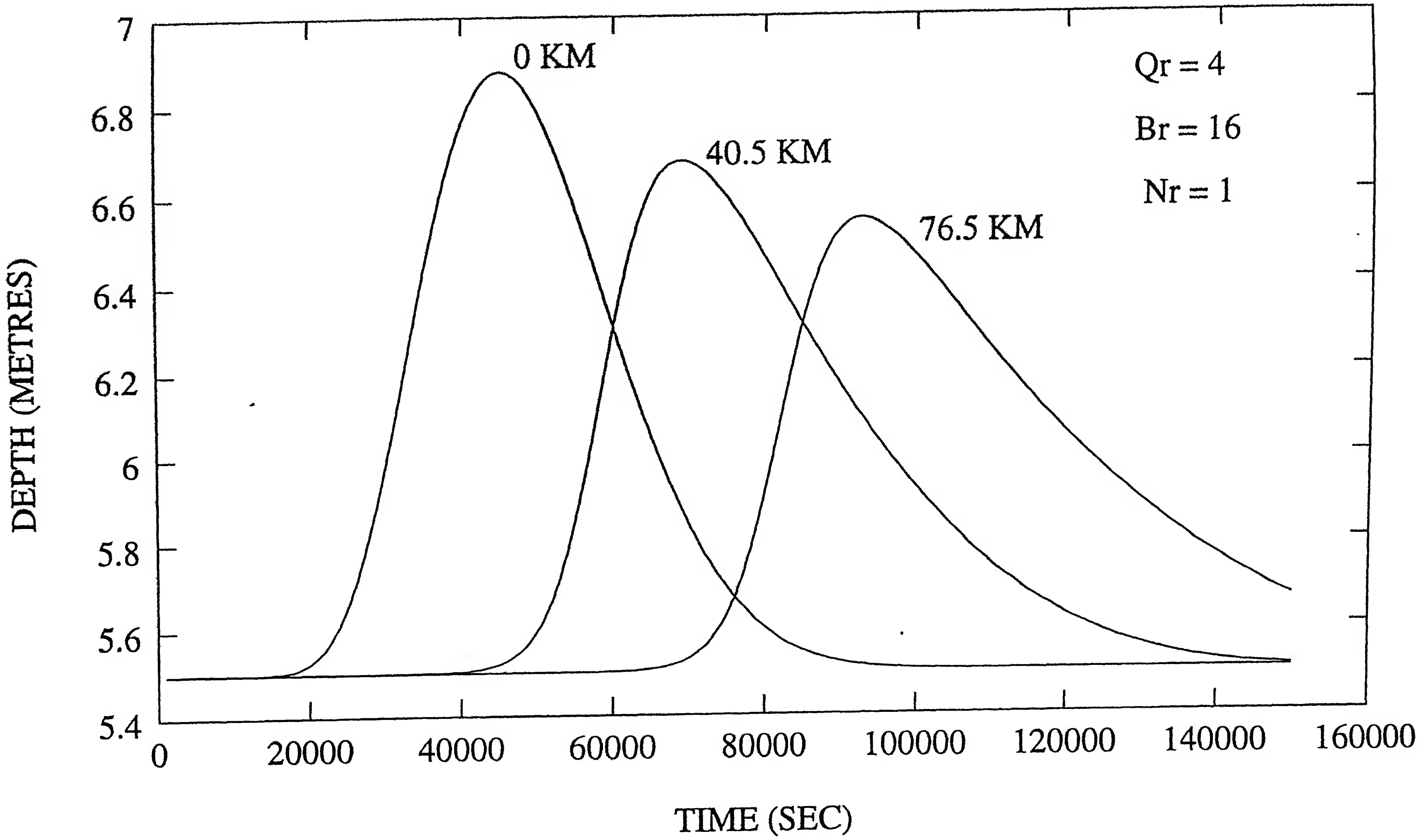


Fig 3.6 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE REIVER

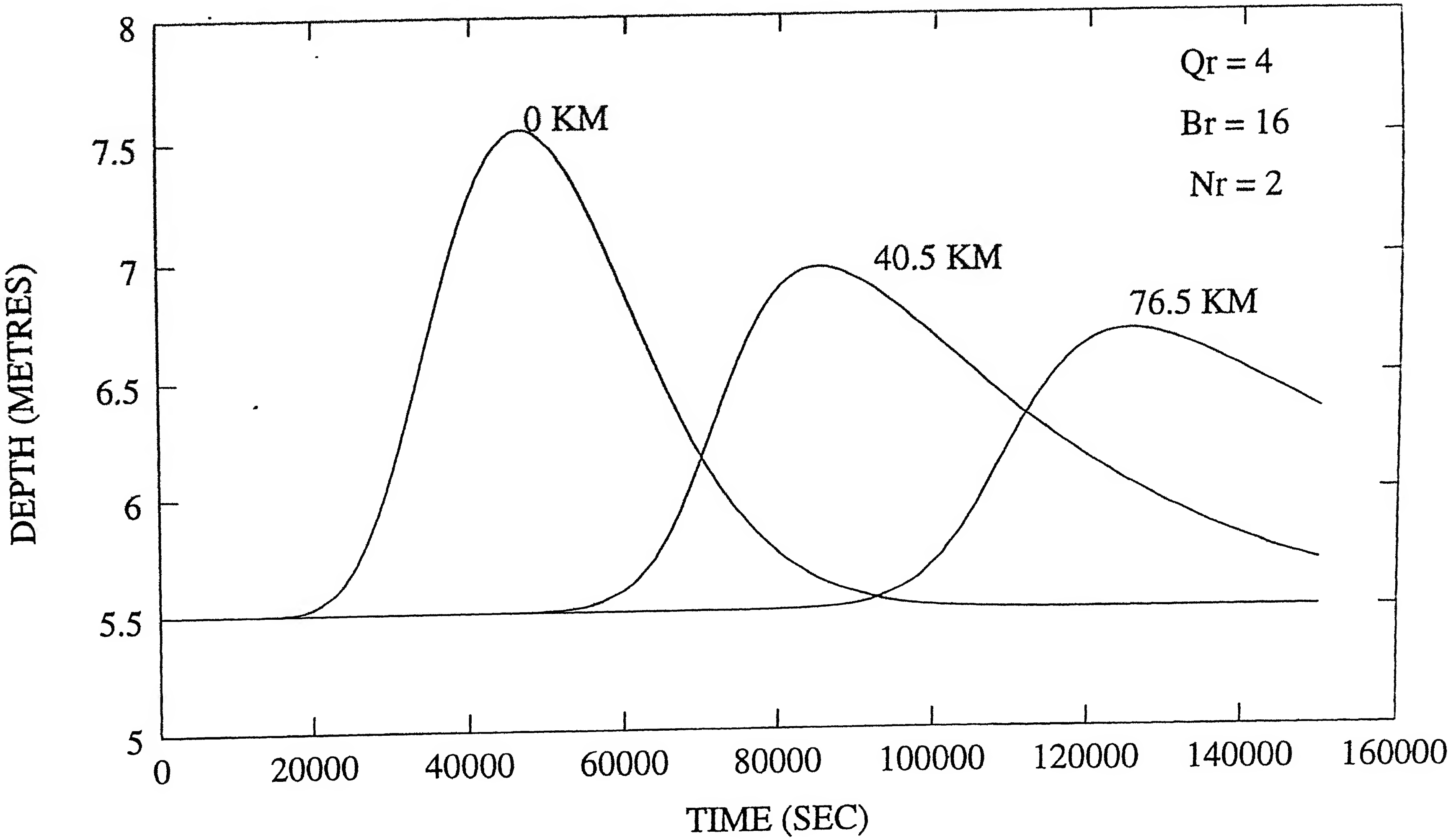


Fig 3.7 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

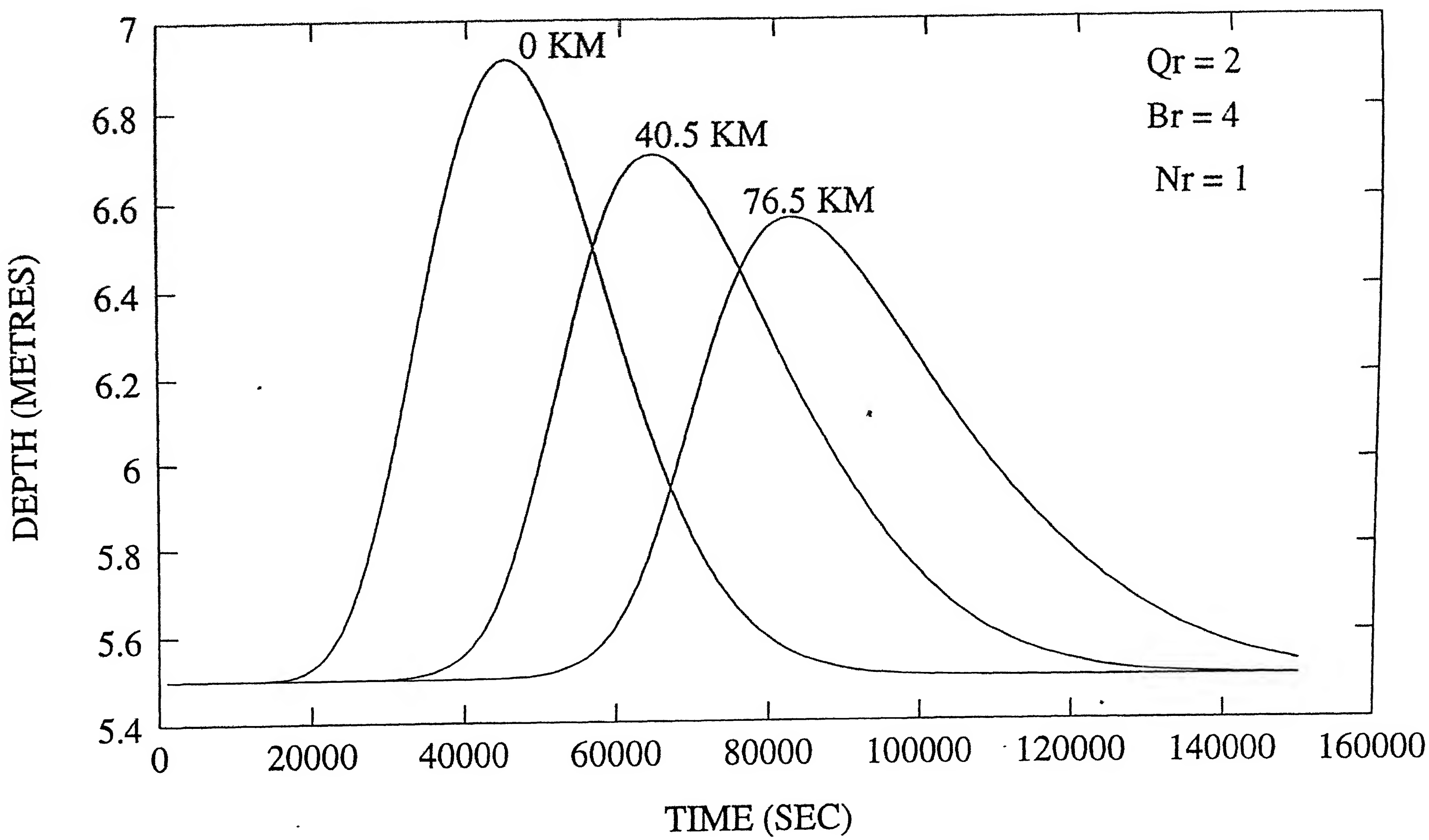


Fig 3.8 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

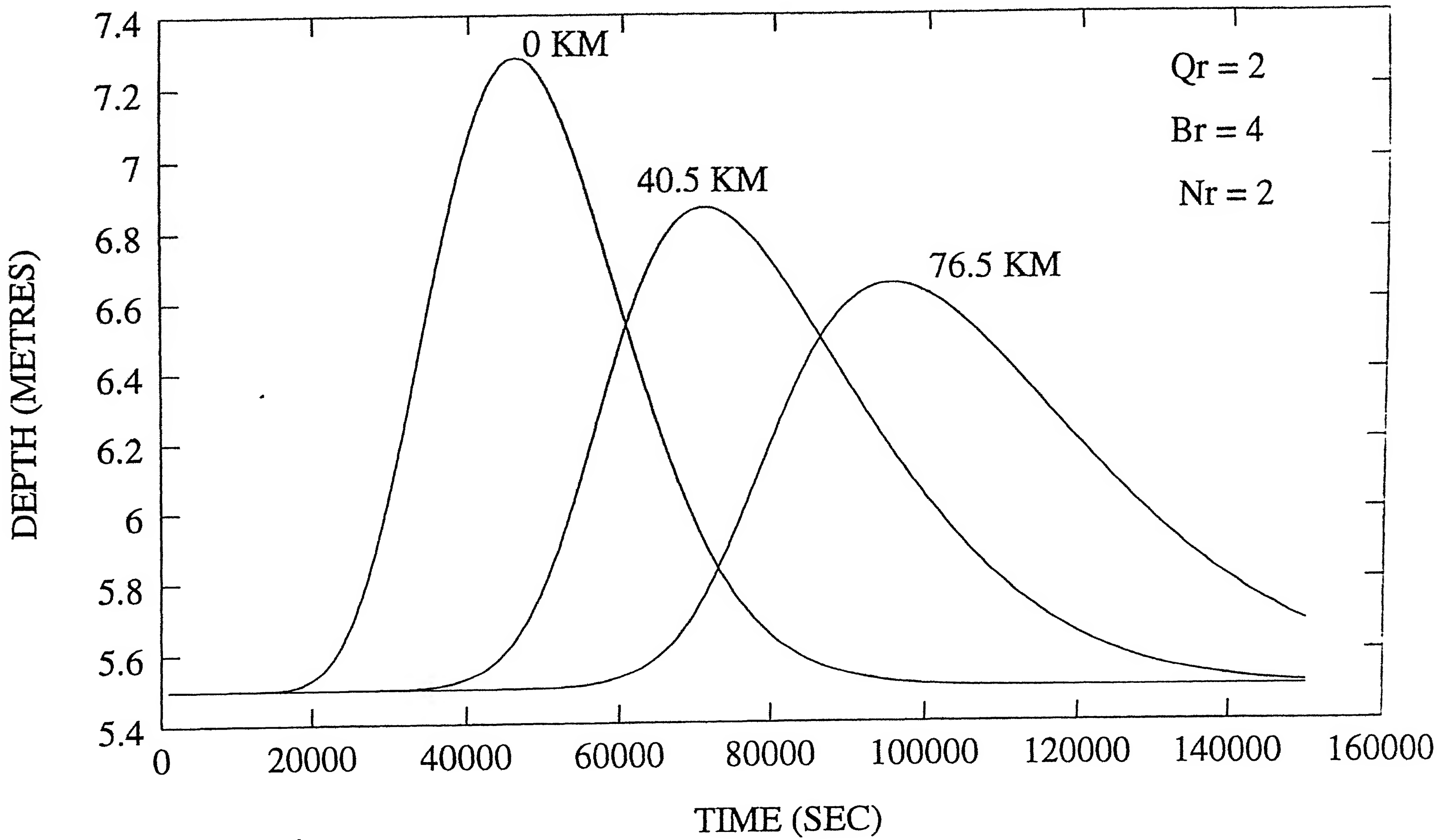


Fig 3.9 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

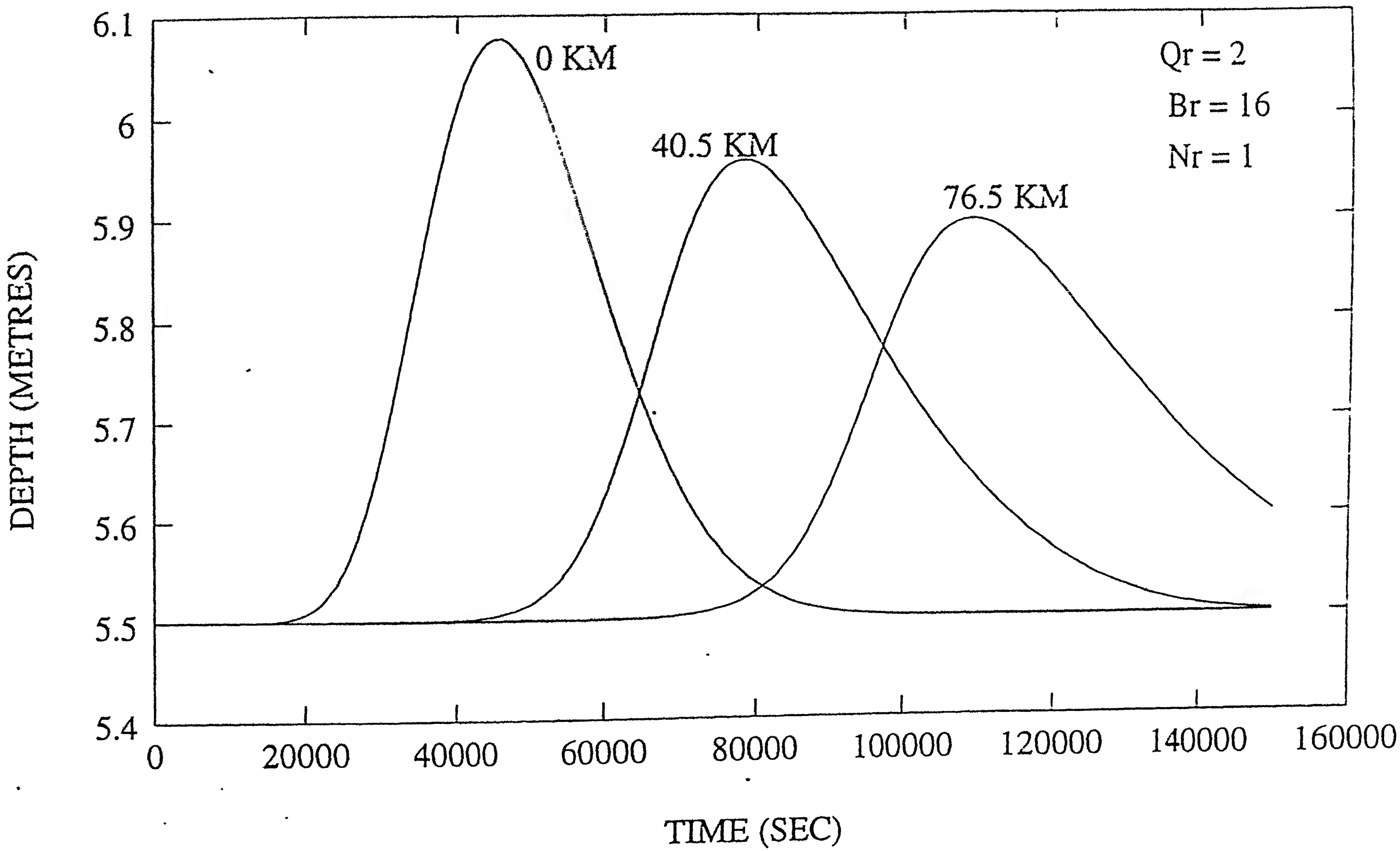


Fig 3.10 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

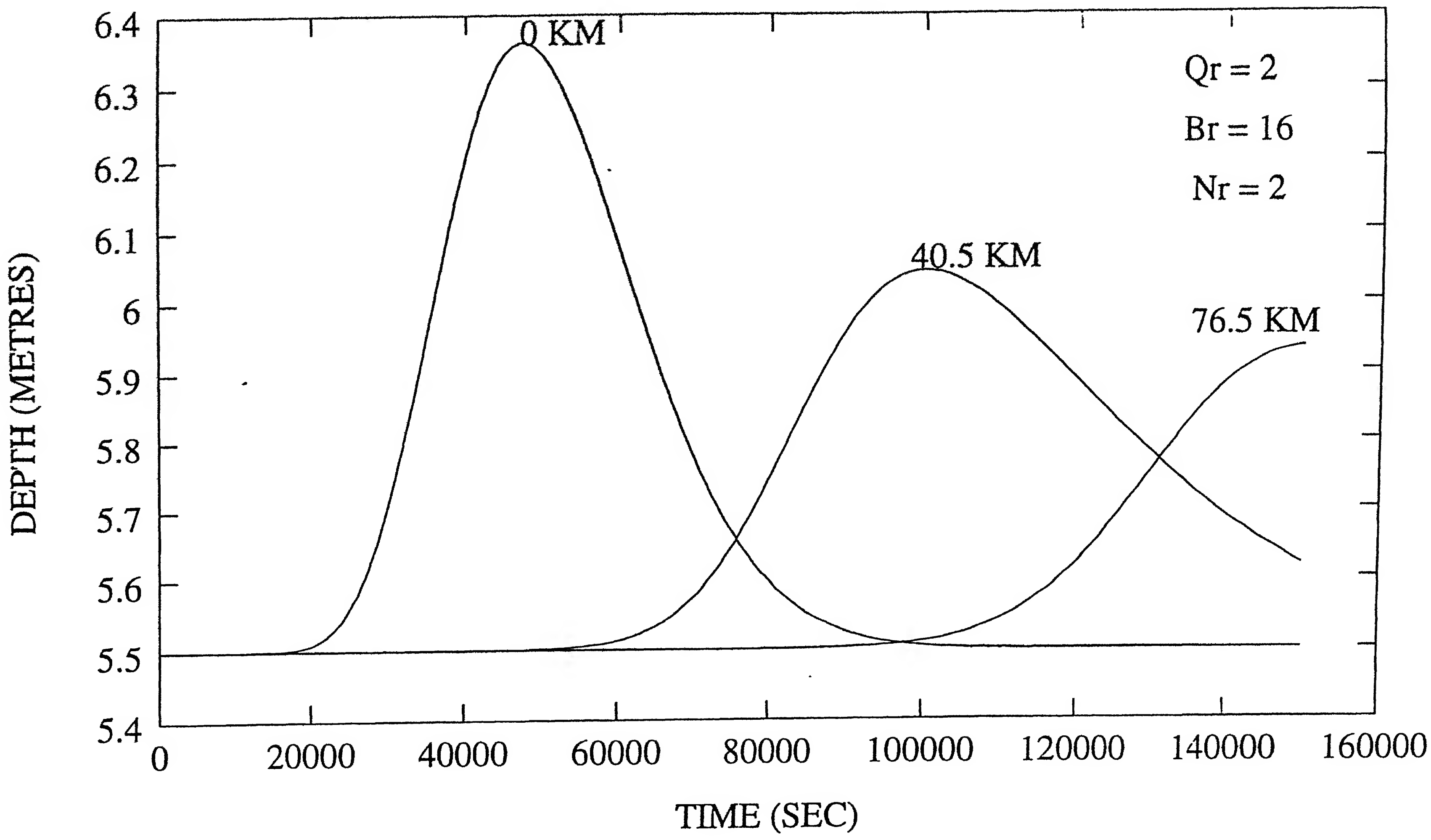


Fig 3.11 STAGE HYDROGRAPH AT DIFFERENT LOCATIONS ALONG THE RIVER

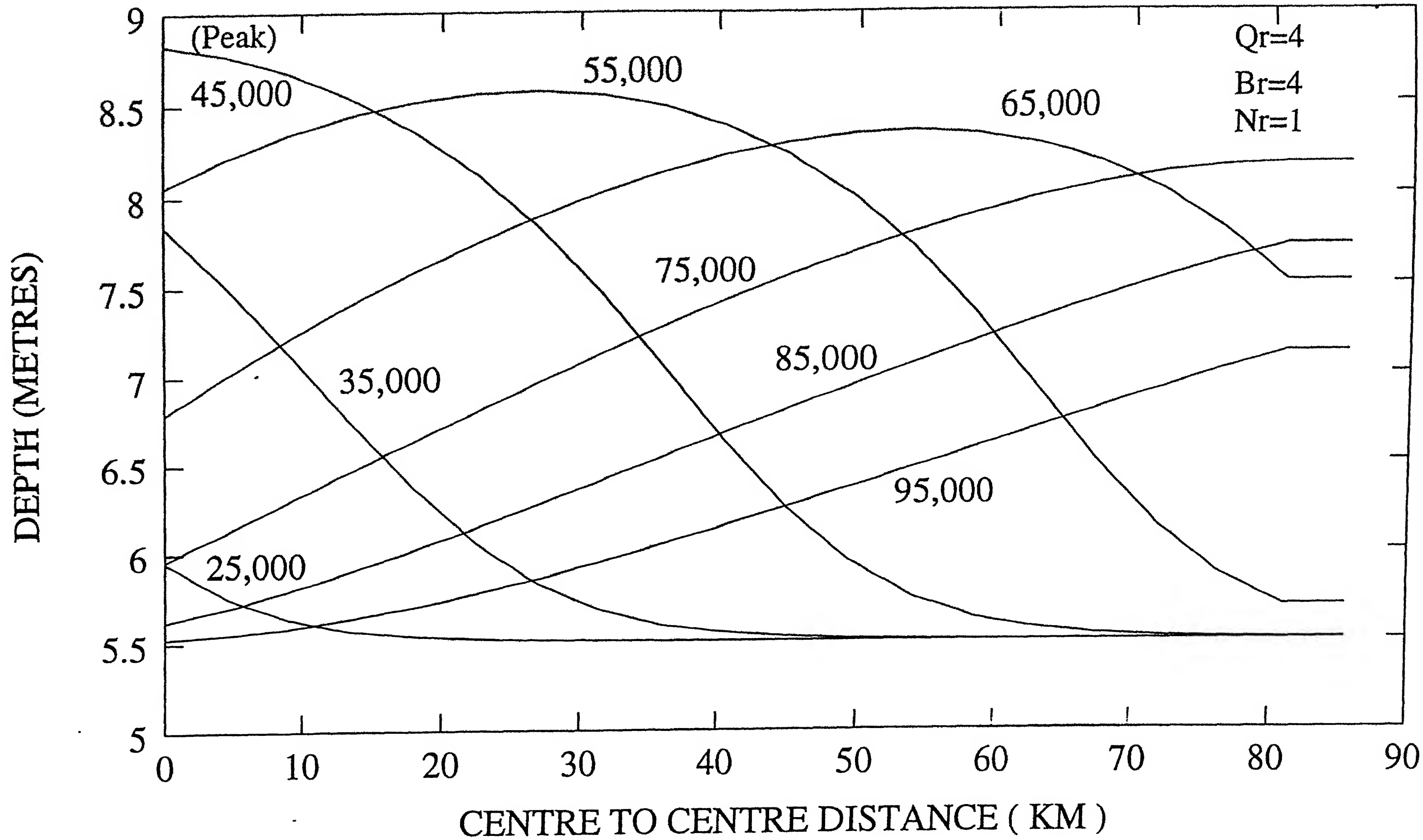


Fig 3.12 · WATER SURFACE PROFILES AT DIFFERENT TIMES(SECONDS) ALONG THE RIVER

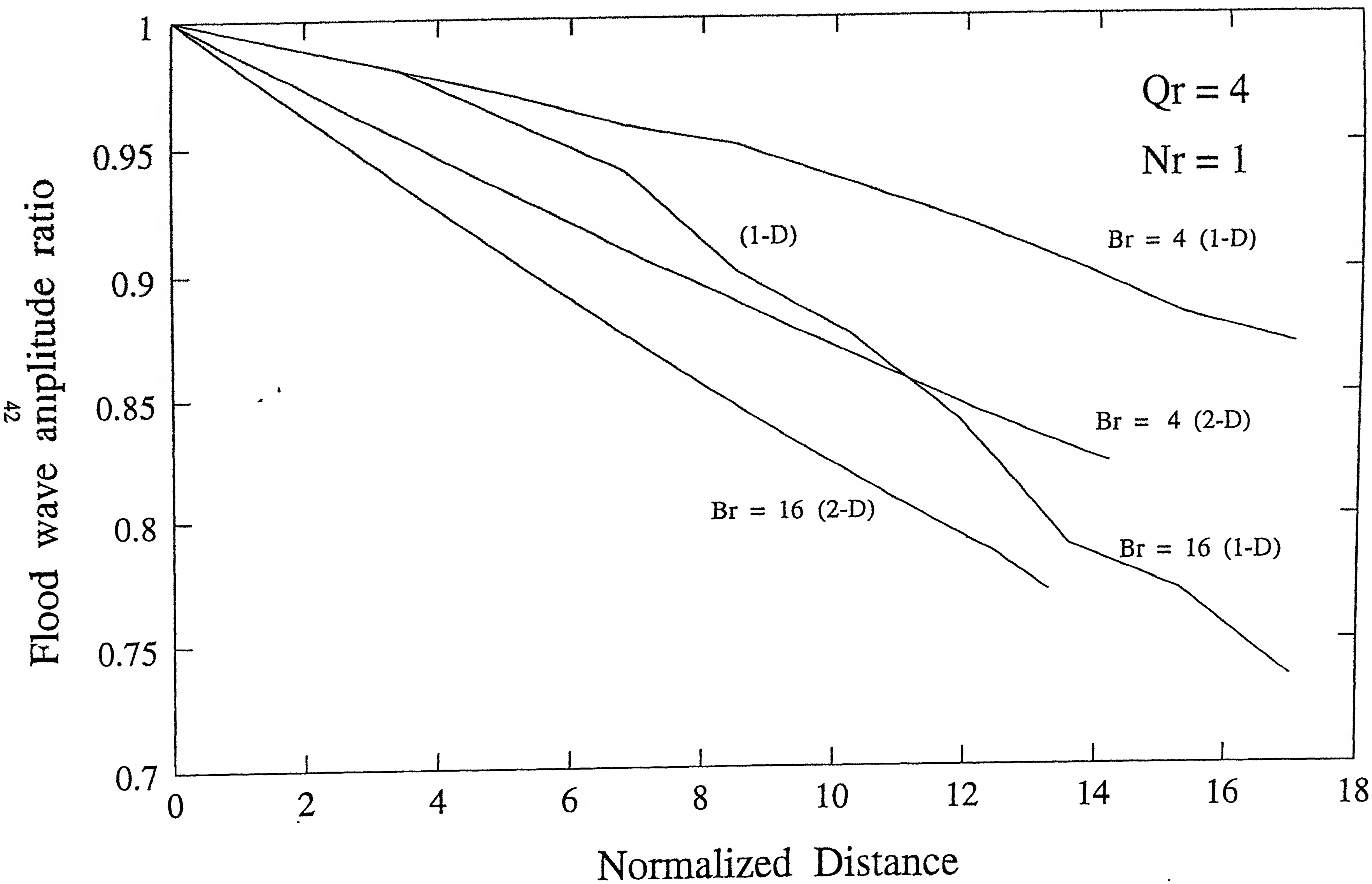


Fig 3.13 Effect of Br on Flood peak subsidence

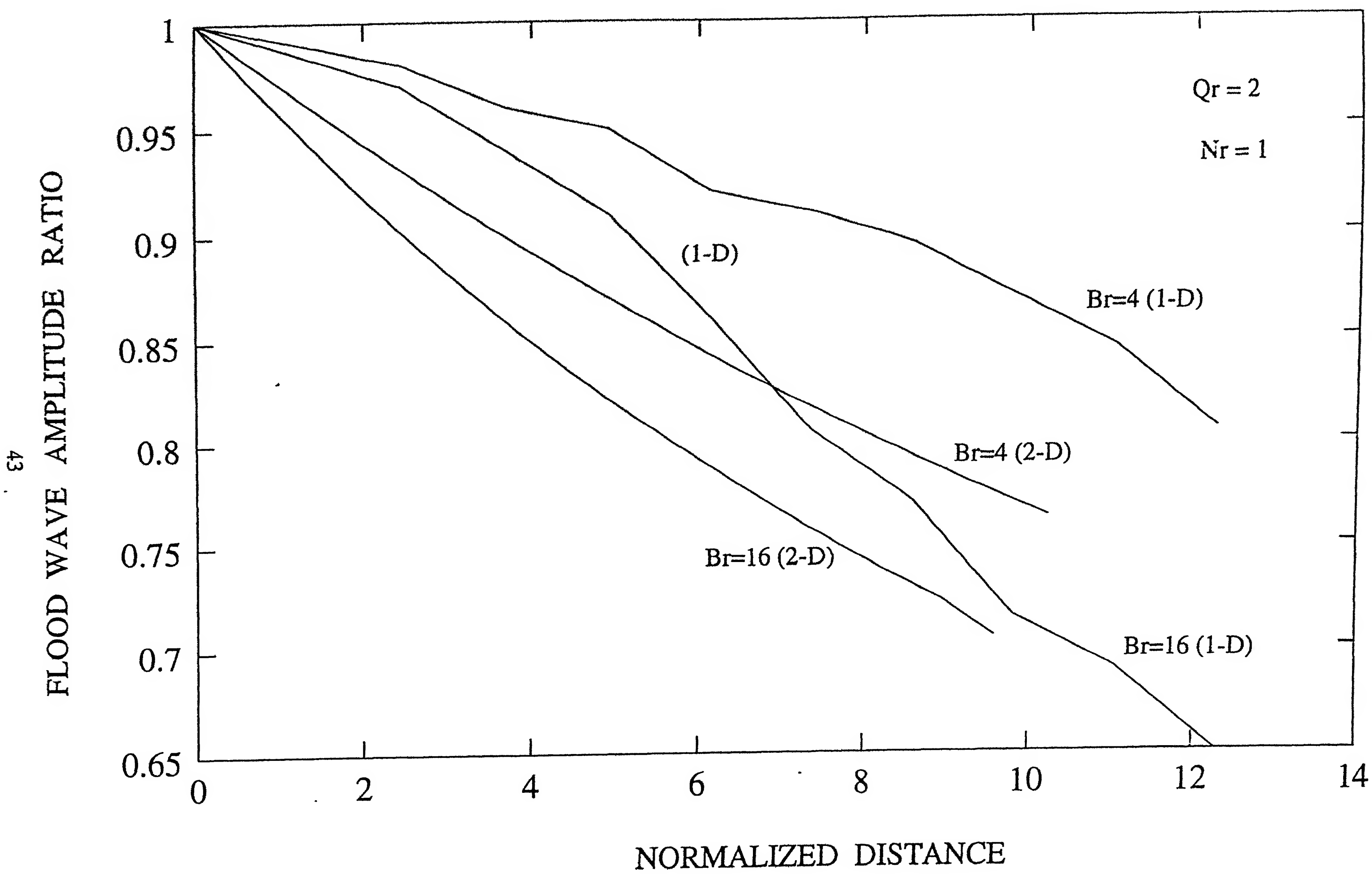


Fig 3.14 Effect of Br on Flood peak subsidence

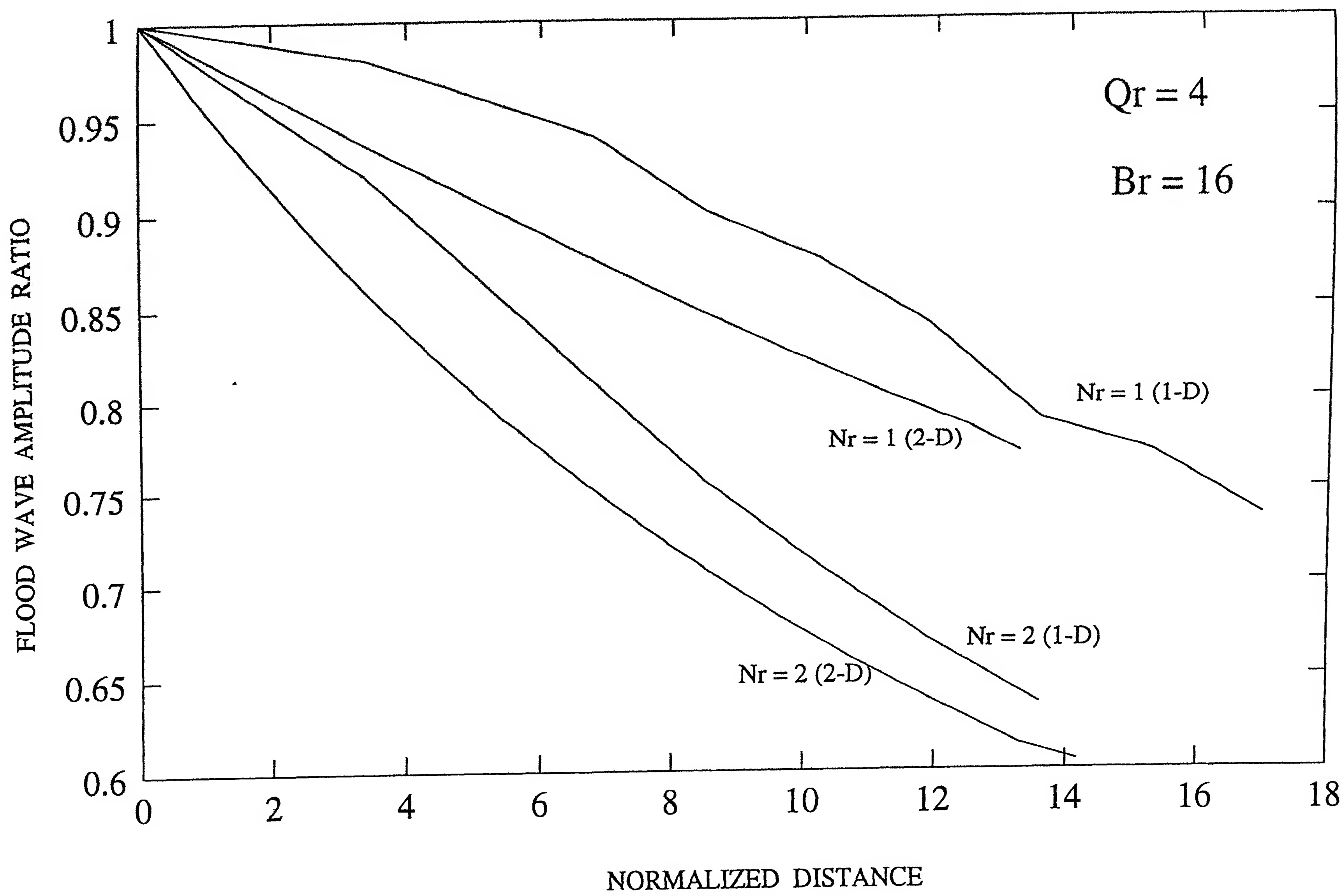


Fig 3.15 Effect of Nr on Flood peak subsidence

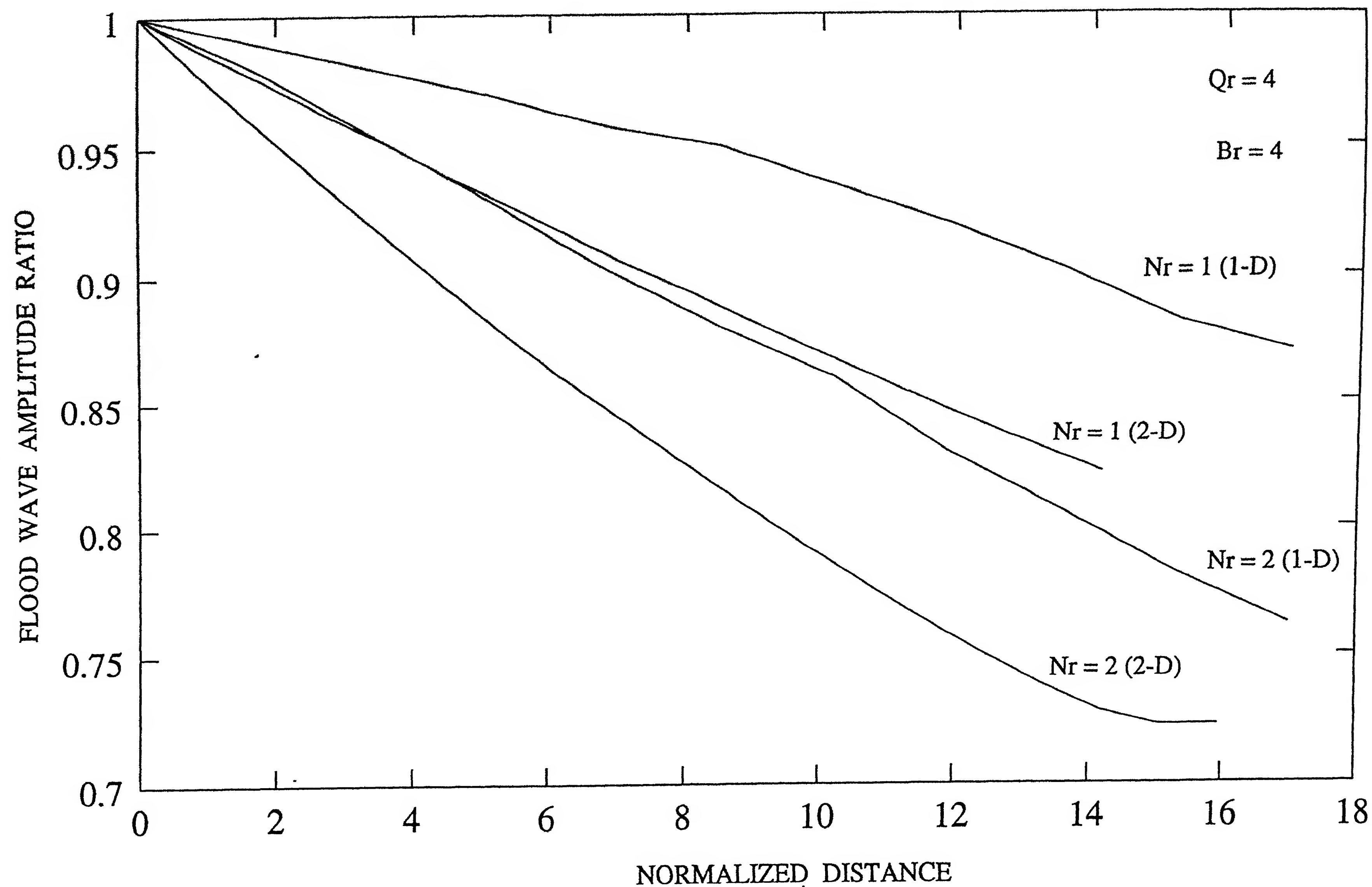


Fig 3.16 Effect of Nr on Flood peak subsidence

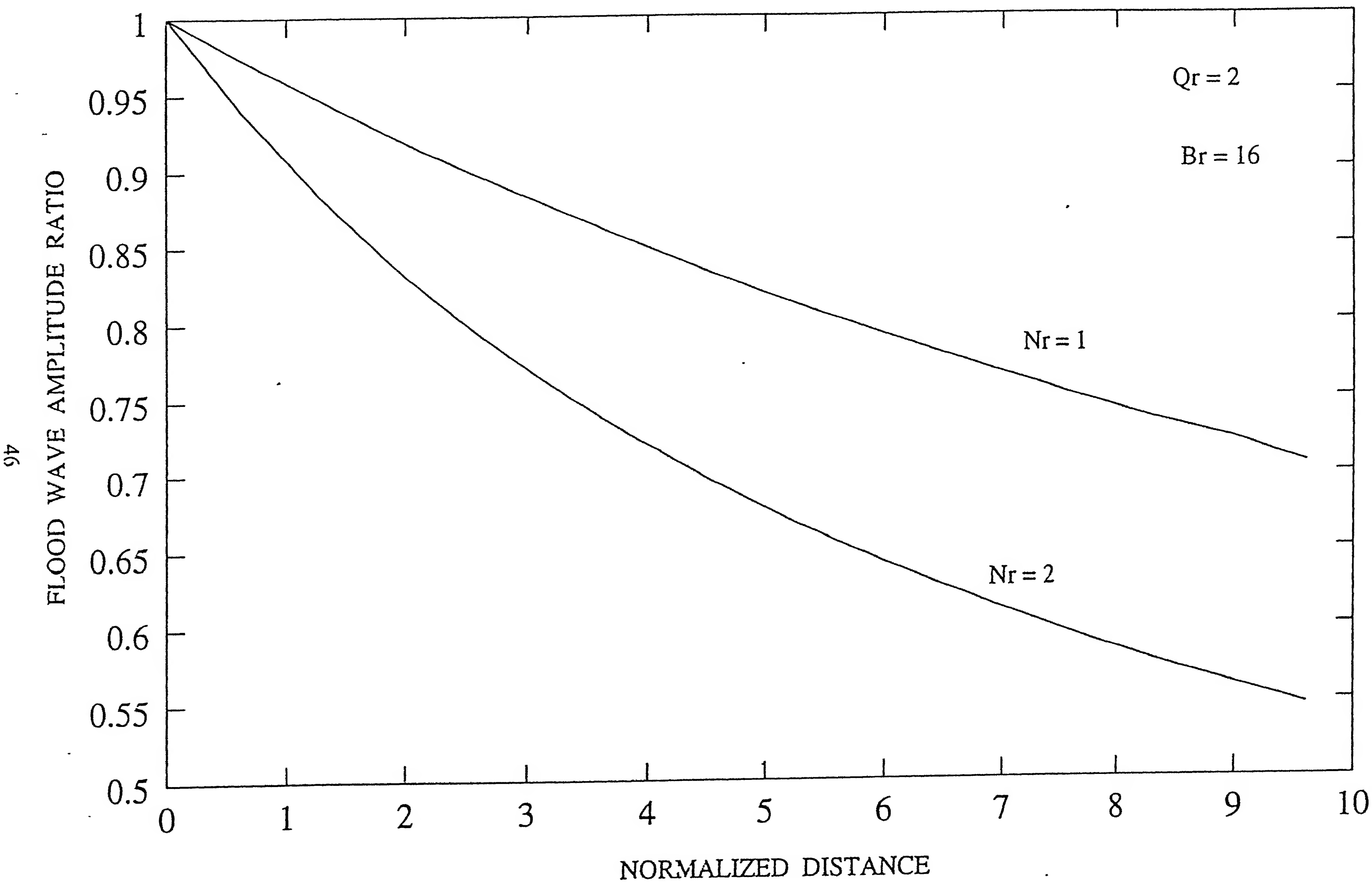


Fig 3.17 Effect of Nr on flood peak subsidence

were obtained in one - dimensional model . Figure 3.18 shows the effect of different sets of parameters on flood peak subsidence .The maximum subsidence was observed for $Q_r = 4$, $B_r = 16$ and $N_r = 2$ and the maximum rate of subsidence was observed for $Q_r = 2$, $B_r = 16$ and $N_r = 2$. The effect of Q_r can be observed from figure 3.18 . Higher values of Q_r result in higher flow depths above the flood plains . Higher flow depths result in more or less uniform flow over the entire cross section . Therefore , higher values of Q_r result in lower rate of subsidence . Figures 3.19 - 3.20 show the variation of parameters for $Q_r = 4$ and $Q_r = 2$ respectively . In figure 3.19 , maximum rate of peak subsidence is observed for $B_r = 16$ and $N_r = 2$ and in figure 3.20 , also the maximum rate of peak subsidence is observed for $B_r = 16$ and $N_r = 2$.

The effect of B_r on the time to reach peak depth at various locations along the river is shown in Figs. 3.21 and 3.22 . In these figures , the normalized distance is shown on the x-axis and the normalized time to reach peak depth is shown on the y-axis . It can be seen that higher the B_r value higher is the time to reach peak depth . Similar results are obtained from one-dimensional model. The storage effects of the flood plains are more for higher values of B_r and therefore , it takes a long time for the peak to travel in the downstream direction .

Stage hydrographs are plotted at 40.5 km from the upstream end in Figs.3.23 and 3.24 to see the effect of N_r on peak depth subsidence .It is clearly seen that higher the N_r value

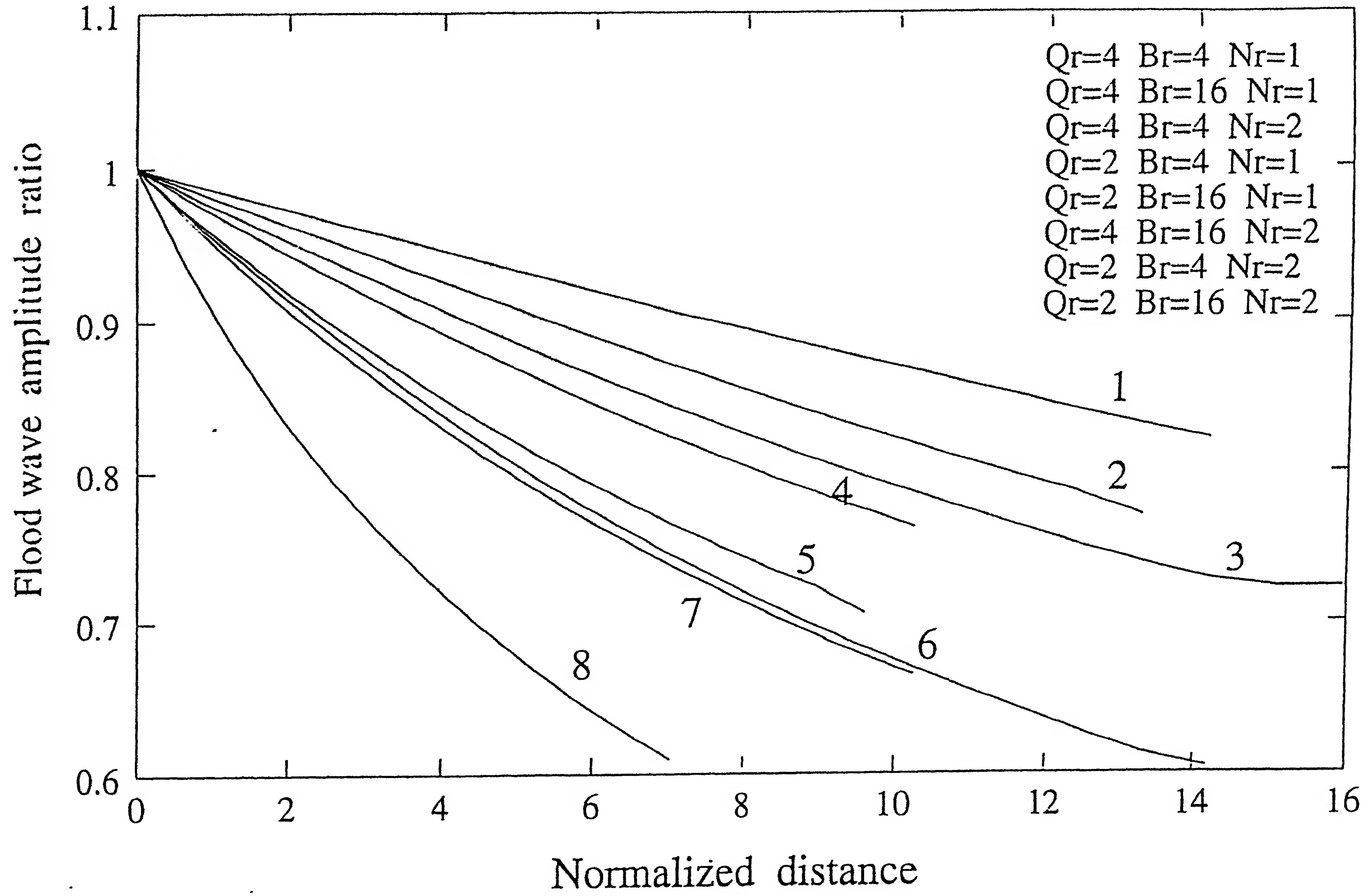


Fig 3:18 Effect of different parameters on Flood peak subsidence along the river

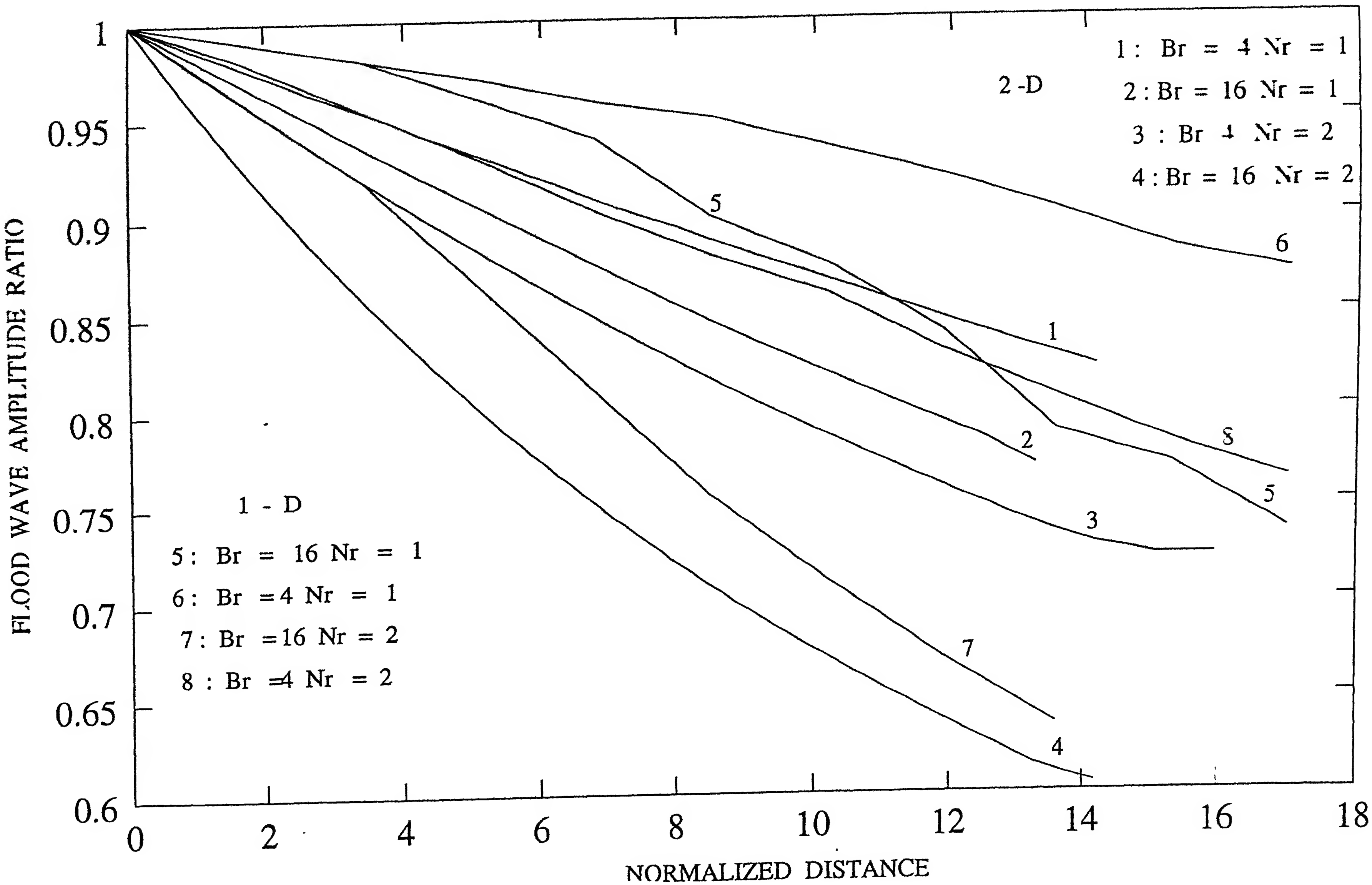


Fig 3.19 Effect of different parameters for Qr=4 on Flood peak subsidence

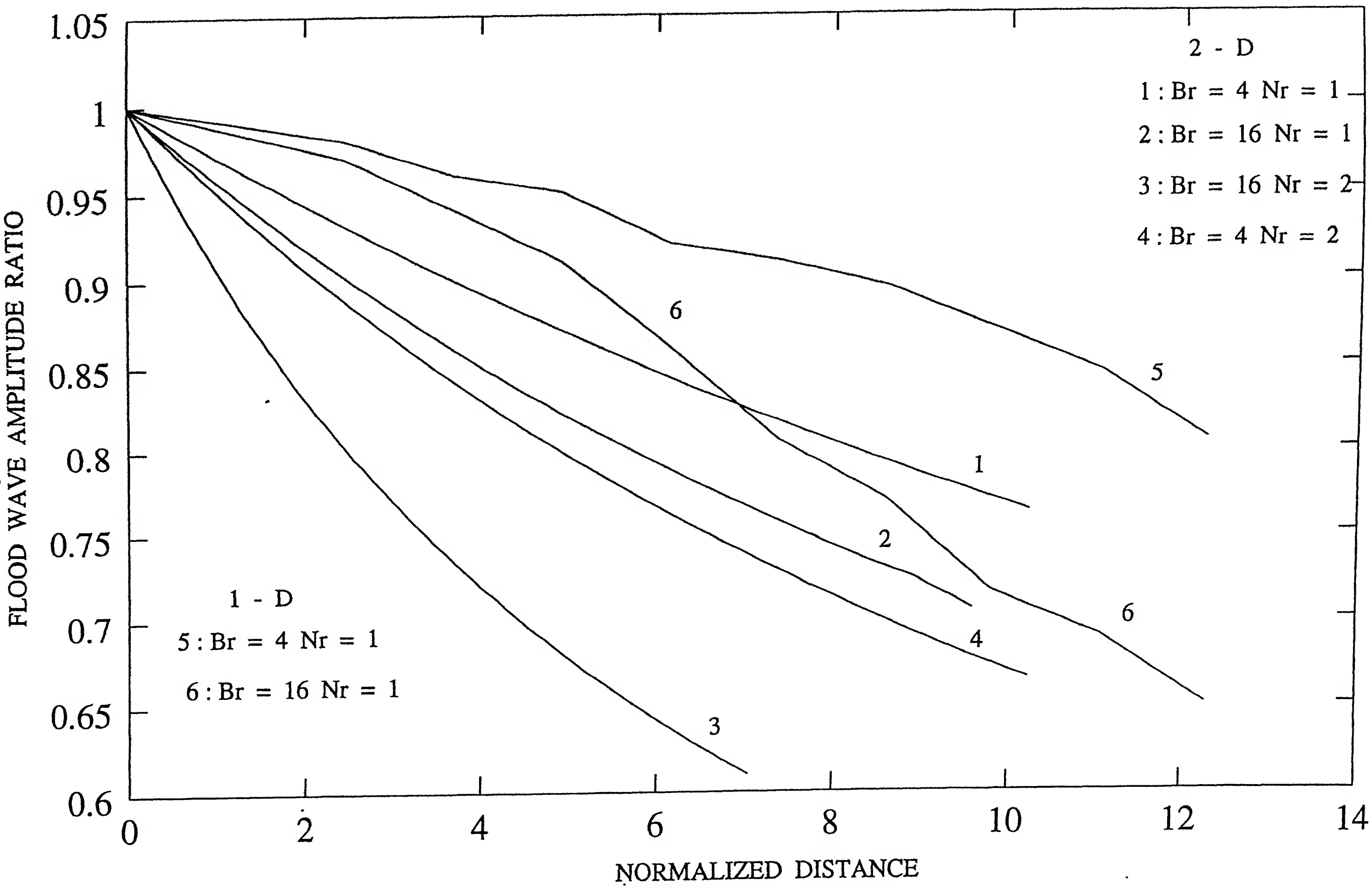


Fig 3.20 Effect of various parameters on Flood peak subsidence for Qr=2

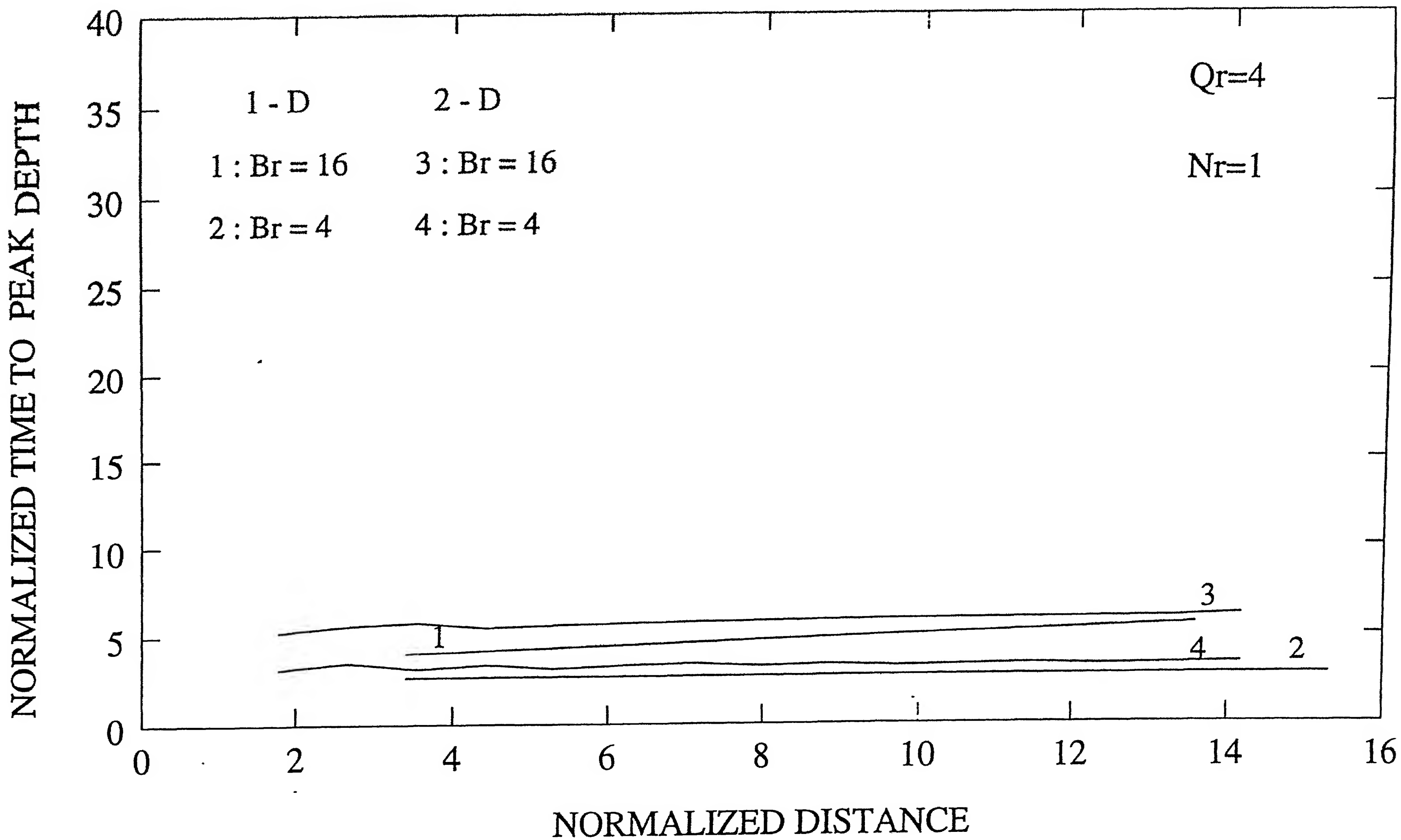


Fig 3.21 TIME TO REACH THE PEAK DEPTH ALONG THE RIVER

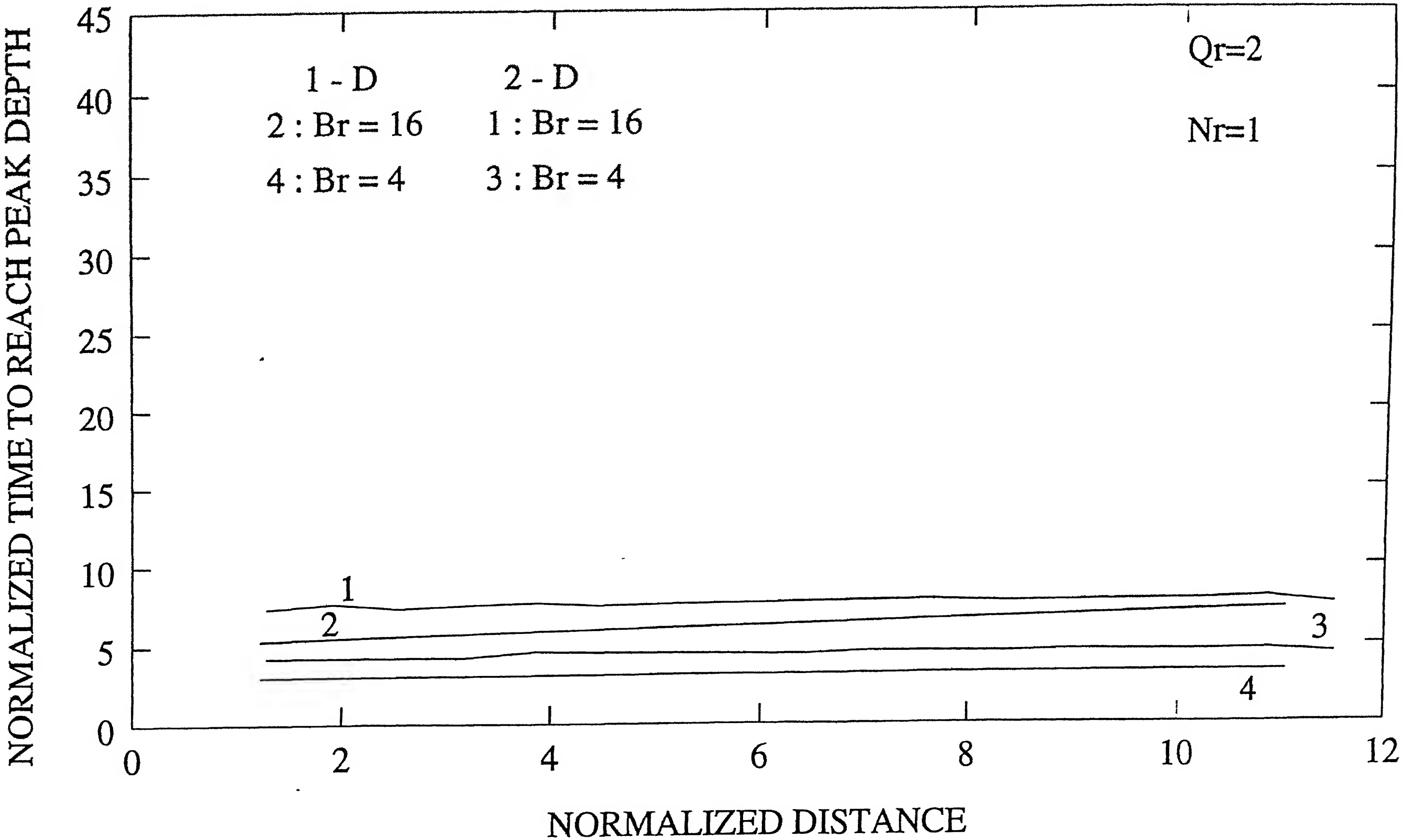


Fig 3.22 TIME TO REACH THE PEAK DEPTH ALONG THE RIVER

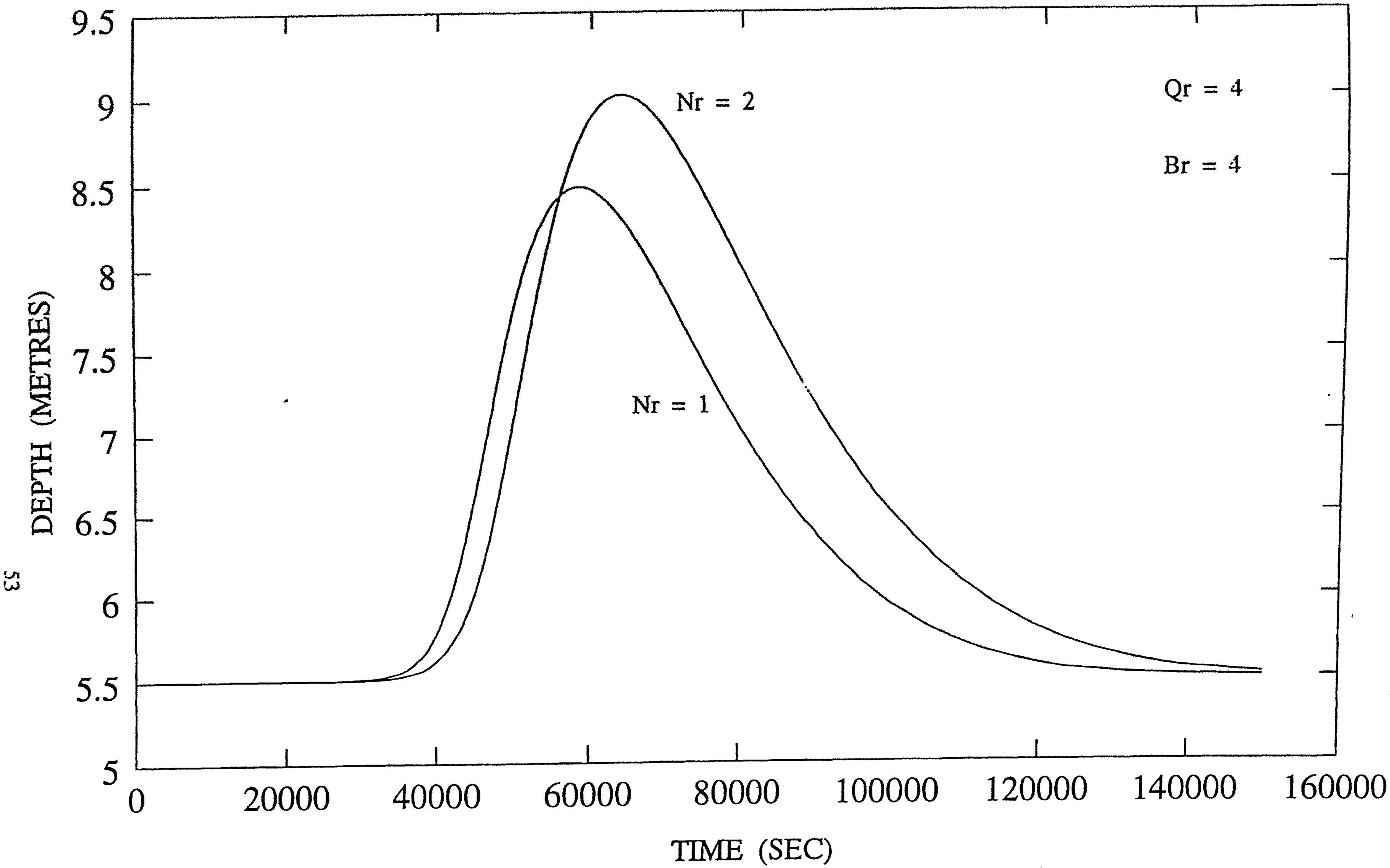


Fig 3.23 Stage hydrograph at 40.5 km centre to centre distance along the river showing the effect of Nr

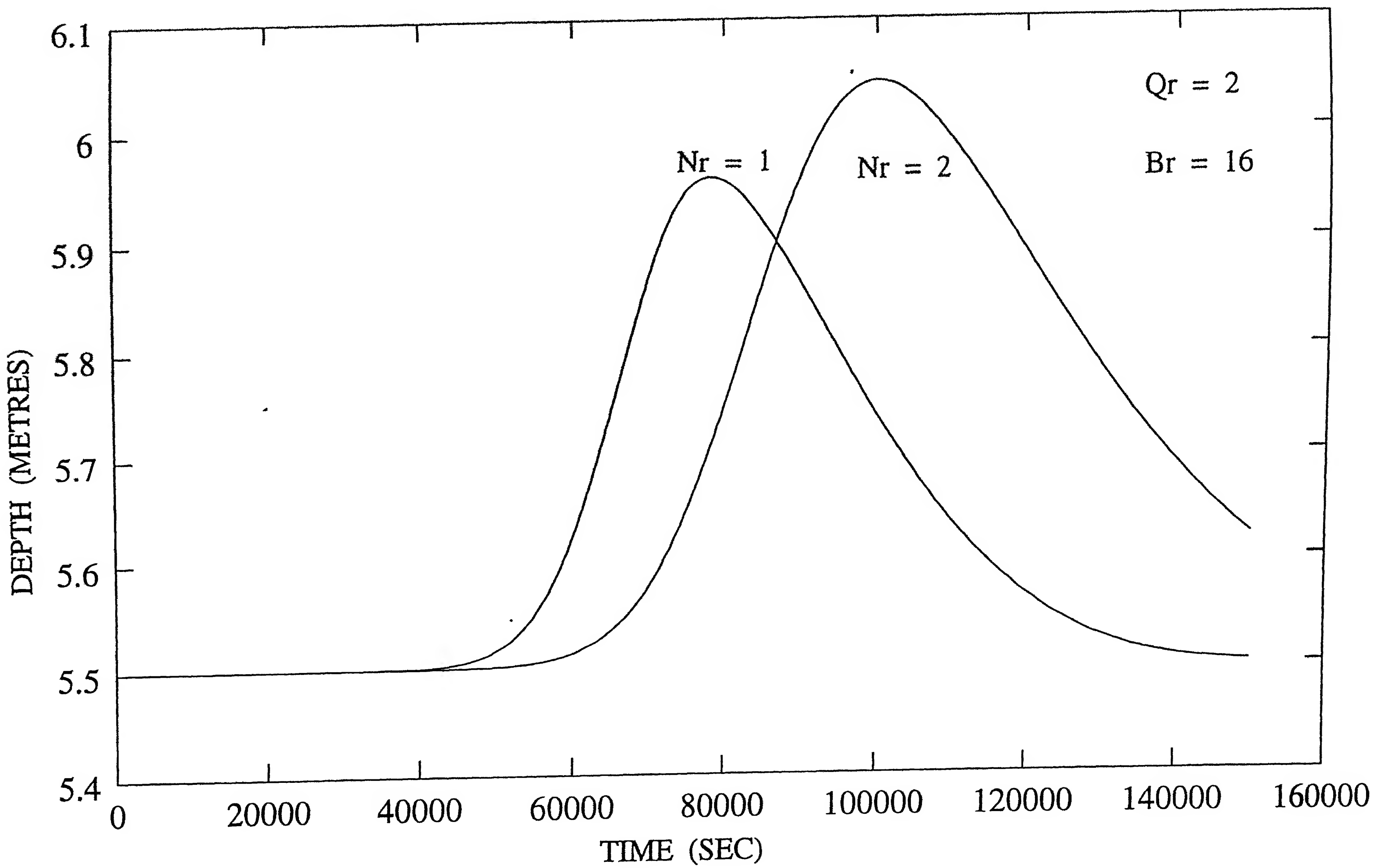


Fig 3.24 Stage hydrograph at 40.5 km from upstream cell measured centre to centre showing the effect of Nr on peak depth

higher is the peak depth and also higher is the time to reach that peak depth . Figures 3.25 and 3.26 show the effect of B_r values on the depth of flow . It is observed that with higher B_r values lower values of peak depths are obtained but the time to reach the peak depth is higher for higher B_r values . Figures 3.27 and 3.28 show the effect of Q_r on the flow depth at 40.5 km station . It is natural to have higher values of peak depths with higher Q_r values and this is observed in these figures .

Water level was found to be almost equal in the flood plain cells and the river cells at any location along the river for any time in all the runs discussed earlier . Computations were also made after decreasing the initial uniform flow depth . A maximum level difference of of 3 cm in the transvers direction was observed (Fig. 3.29 a) when the initial flow depth over the flood plains was 1 cm . Water level difference in the transverse direction was 6 cm (Fig. 3.29 b) when the initial uniform flow depth over the flood plains was 1 mm . Further decrease of initial flow depth does not cause any more difference than 6 cm as shown in figures 3.30 (a) and 3.30 (b) . In figures 3.29 and 3.30 the transverse section is taken at 40.5 km from upstream end . It is also observed from the computations (Figs. 3.31 a and 3.31 b) that as the flow depth in the flood plains become equal to 10 cm , the difference in the water levels along the transverse direction reduced to zero at any location along the river . Due to this reason , the transverse profiles in the figures 3.29 and 3.30 were drawn at 70,000 sec and not at the peak stage . For the case of initial flow depth equal to

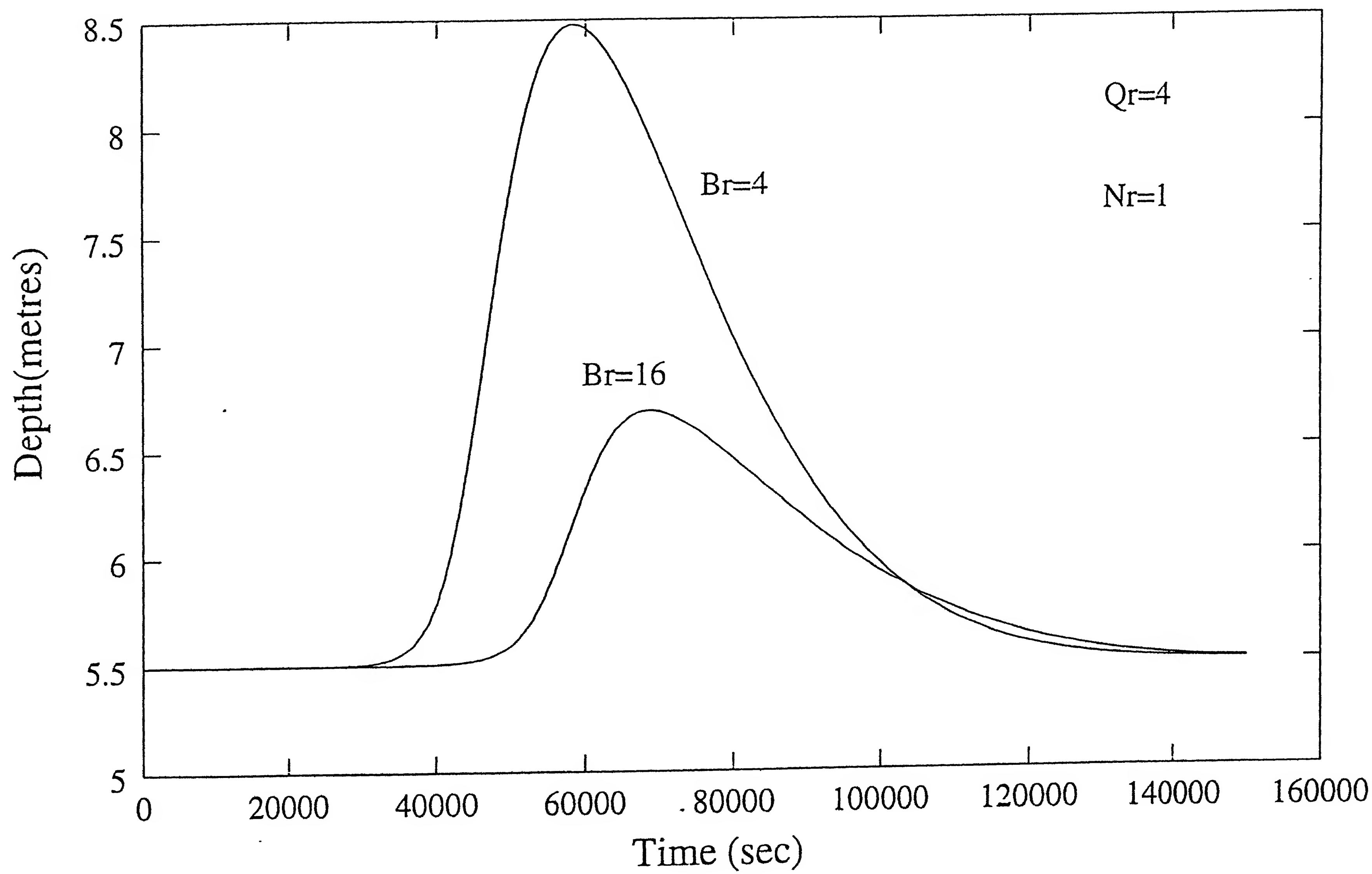


Fig 3.25 Stage hydrograph at 40.5 km from upstream cell showing the effect of Br on peak depth

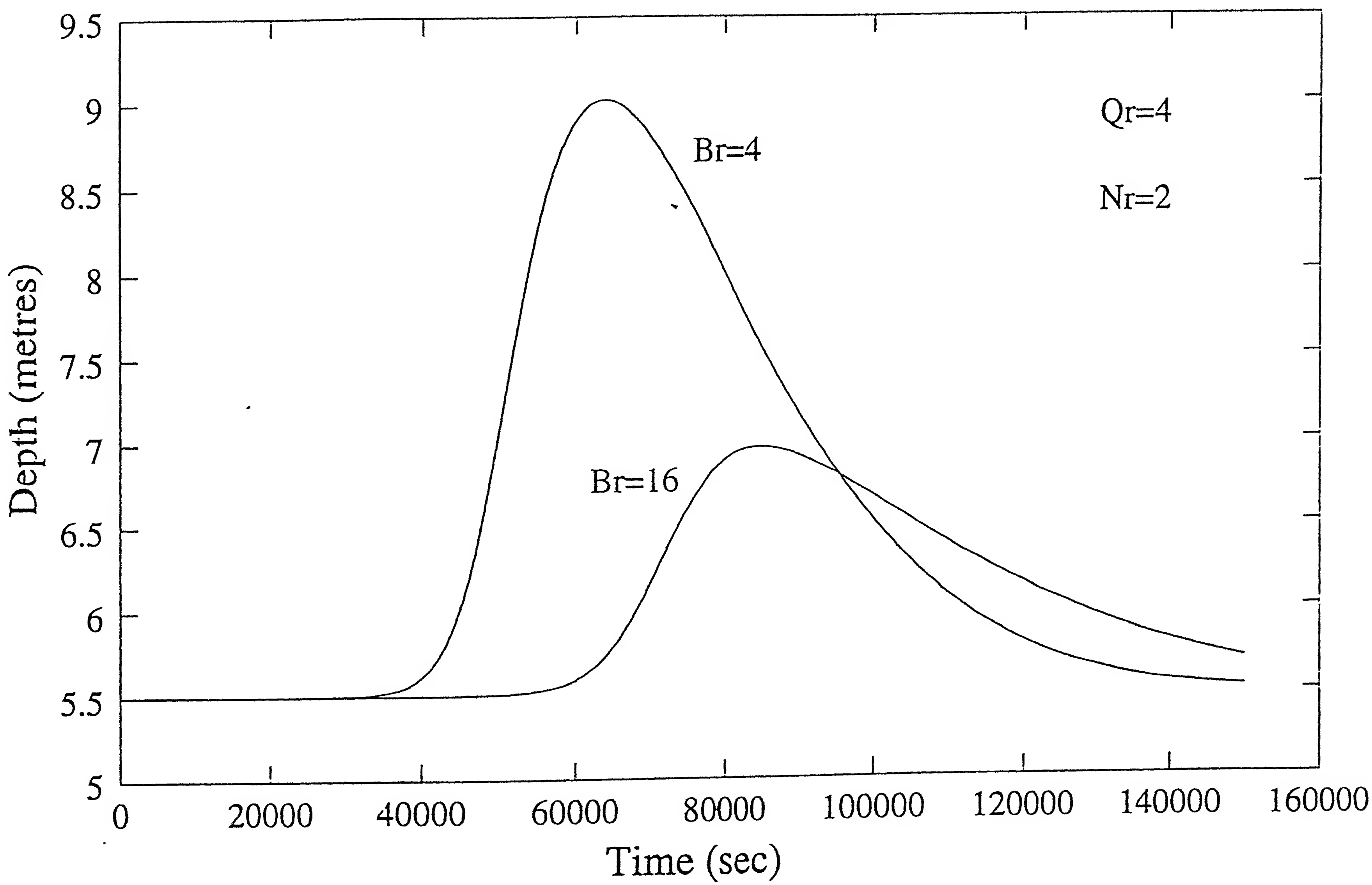


Fig 3.26 Stage hydrograph at 40.5 km from upstream cell along the river showing the effect of Br on peak depth

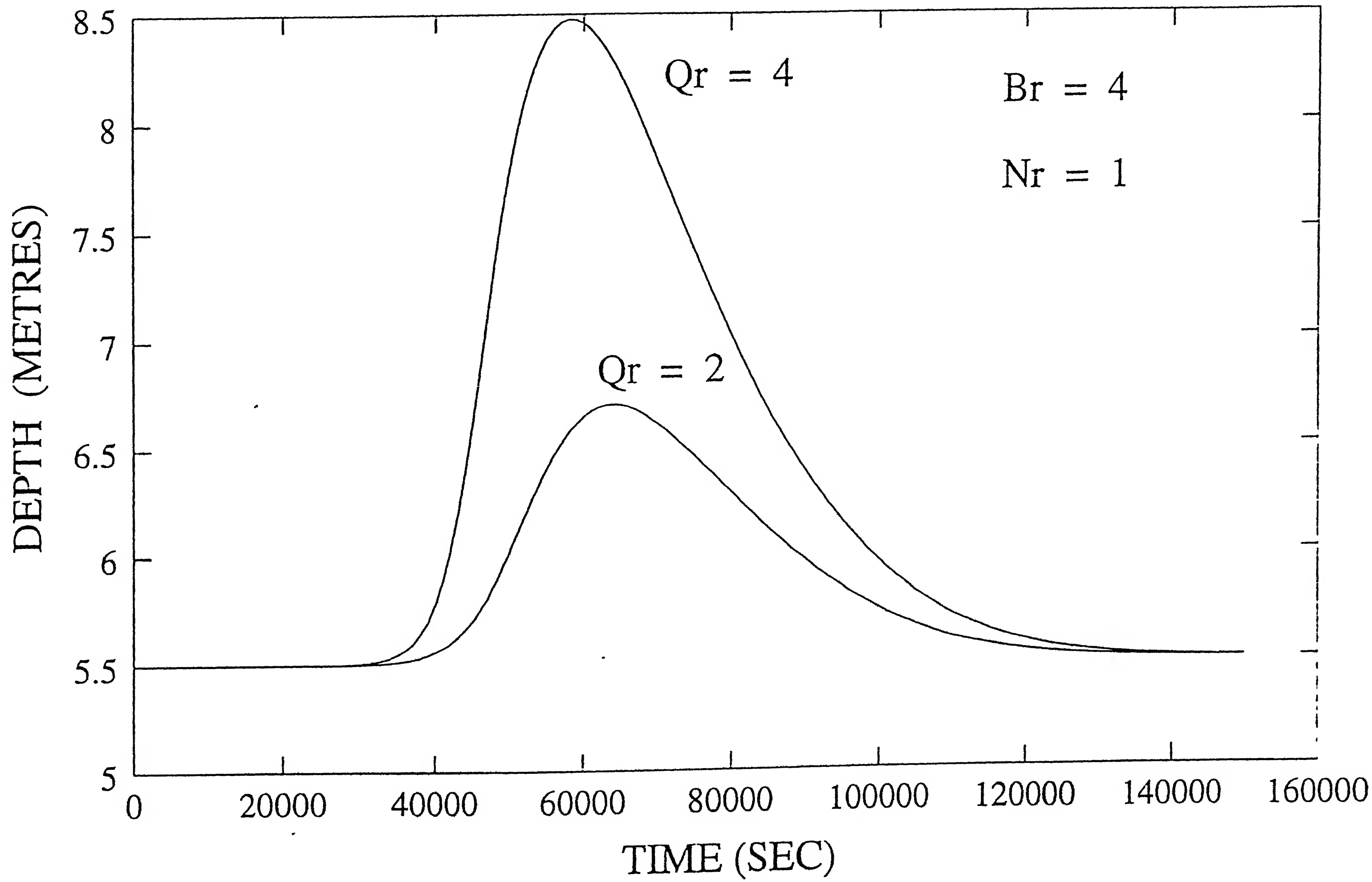


Fig 3.27 stage hydrograph at 40.5 km from upstream cell showing the effect of Qr on peak depth

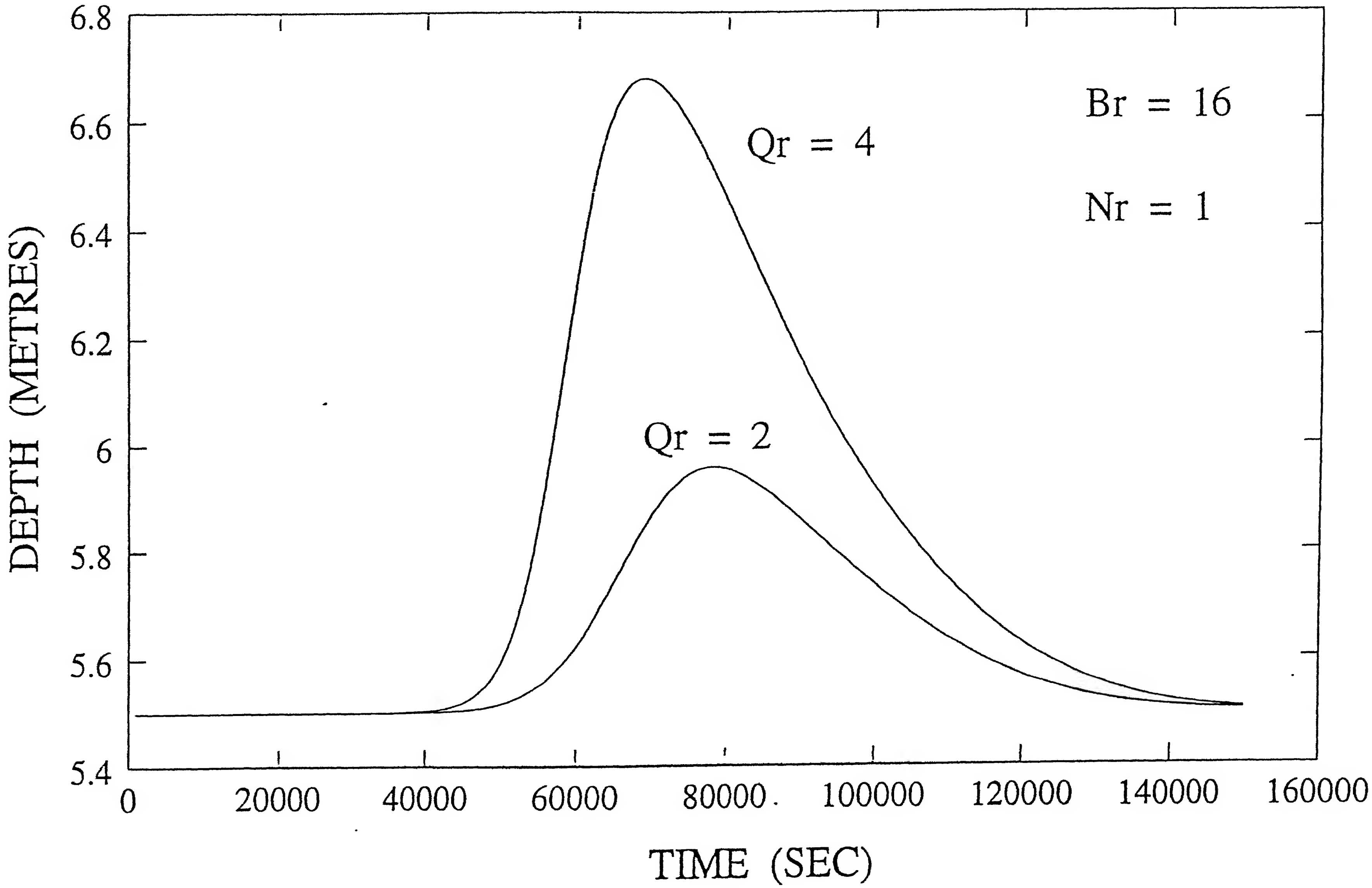
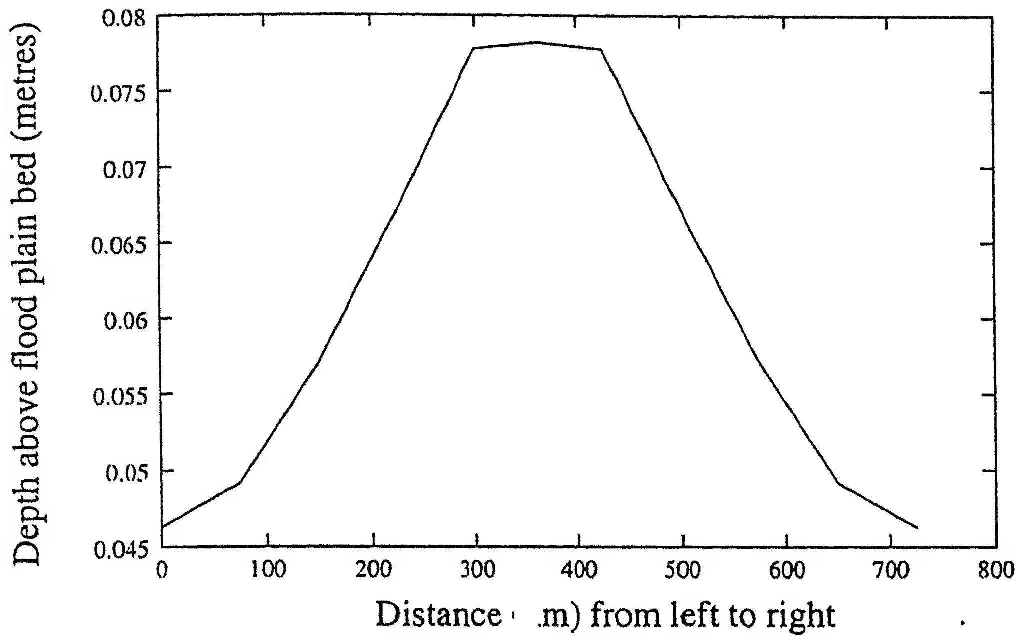
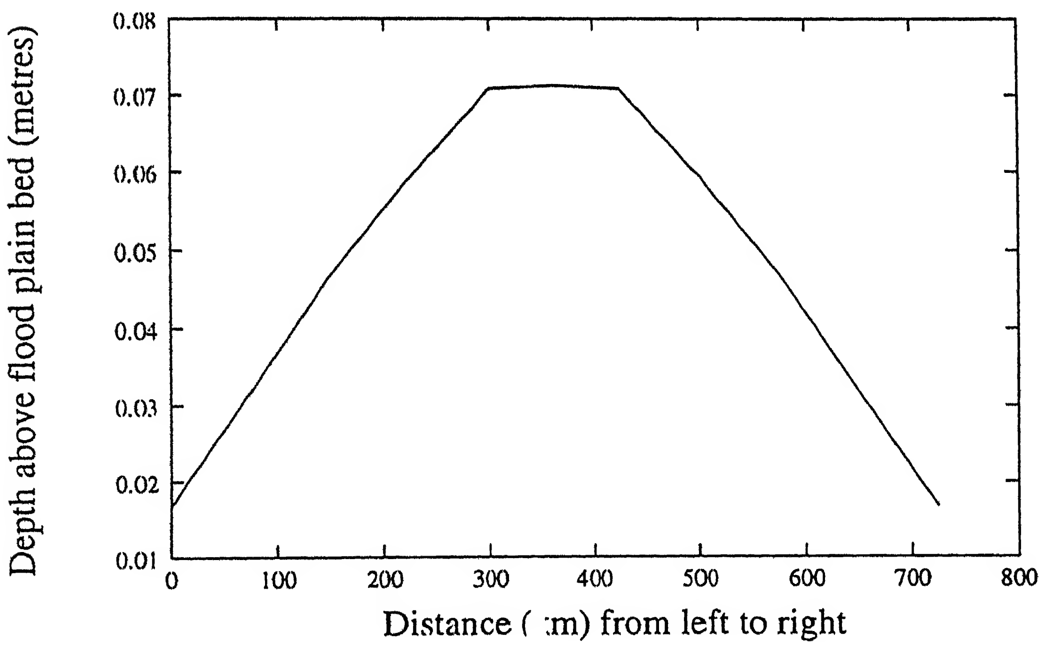


Fig 3.28 Stage hydrograph at 40.5 km from upstream cell showing the effect of Qr on peak depth



(a) with initial uniform flow depth of 1cm above flood plain



(b) with initial uniform flow depth of 1mm above flood plain

Fig 3.29 TRANSVERSE PROFILE AT 40.5 KM FROM UPSTREAM CELL Qr=4 Br=16 Nr=2

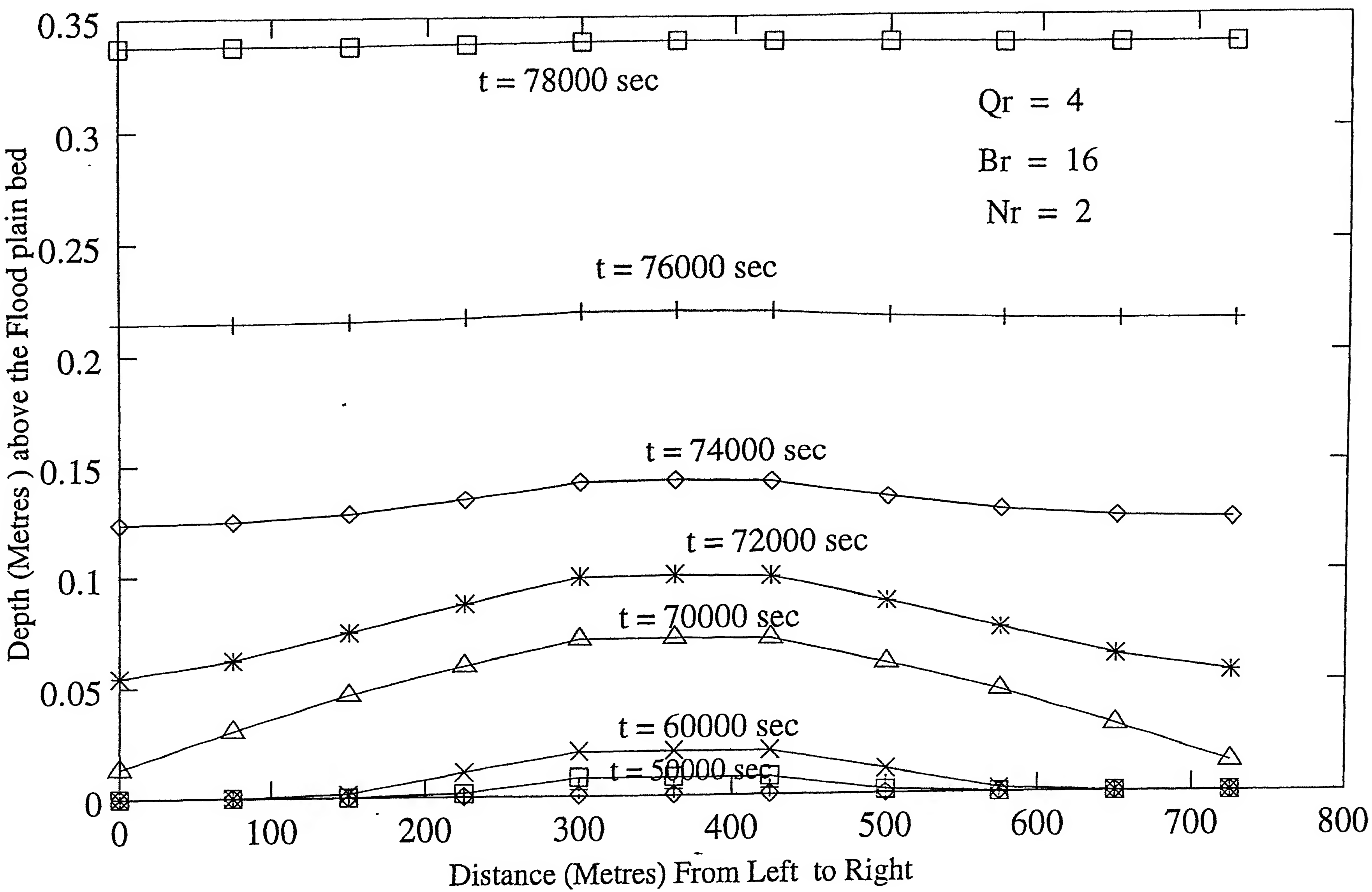


Fig 3.3 i (a) Transverse profile at 40.5 km from upstream cell at different time levels for 5.0001 initial flow depth

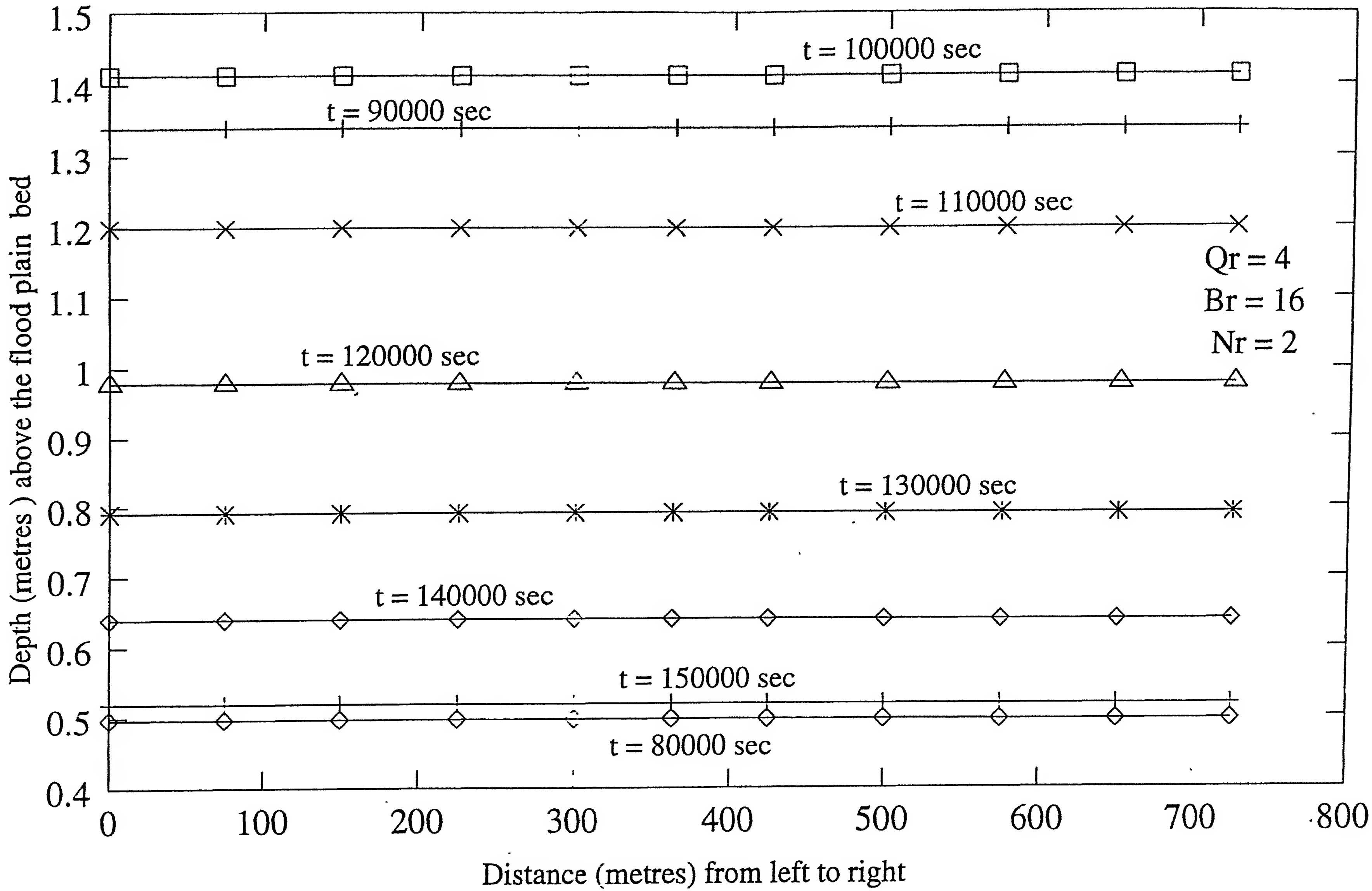


Fig 3.3i (b) Transverse profile at 40.5 km from the upstream cells along the river at various times for initial flow depth of 5.0001 metres .

be 5.5 m , the water level difference in the transverse direction was observed to be almost equal to zero . The effect of initial flow depth on the flood peak subsidence is shown in Fig. 3.32 . Lower initial flow depths introduce significant two-dimensional effects and consequently increase the rate of flood peak subsidence. The two - dimensional effect on the peak depth translation in the downstream direction is shown in Fig. 3.33 .

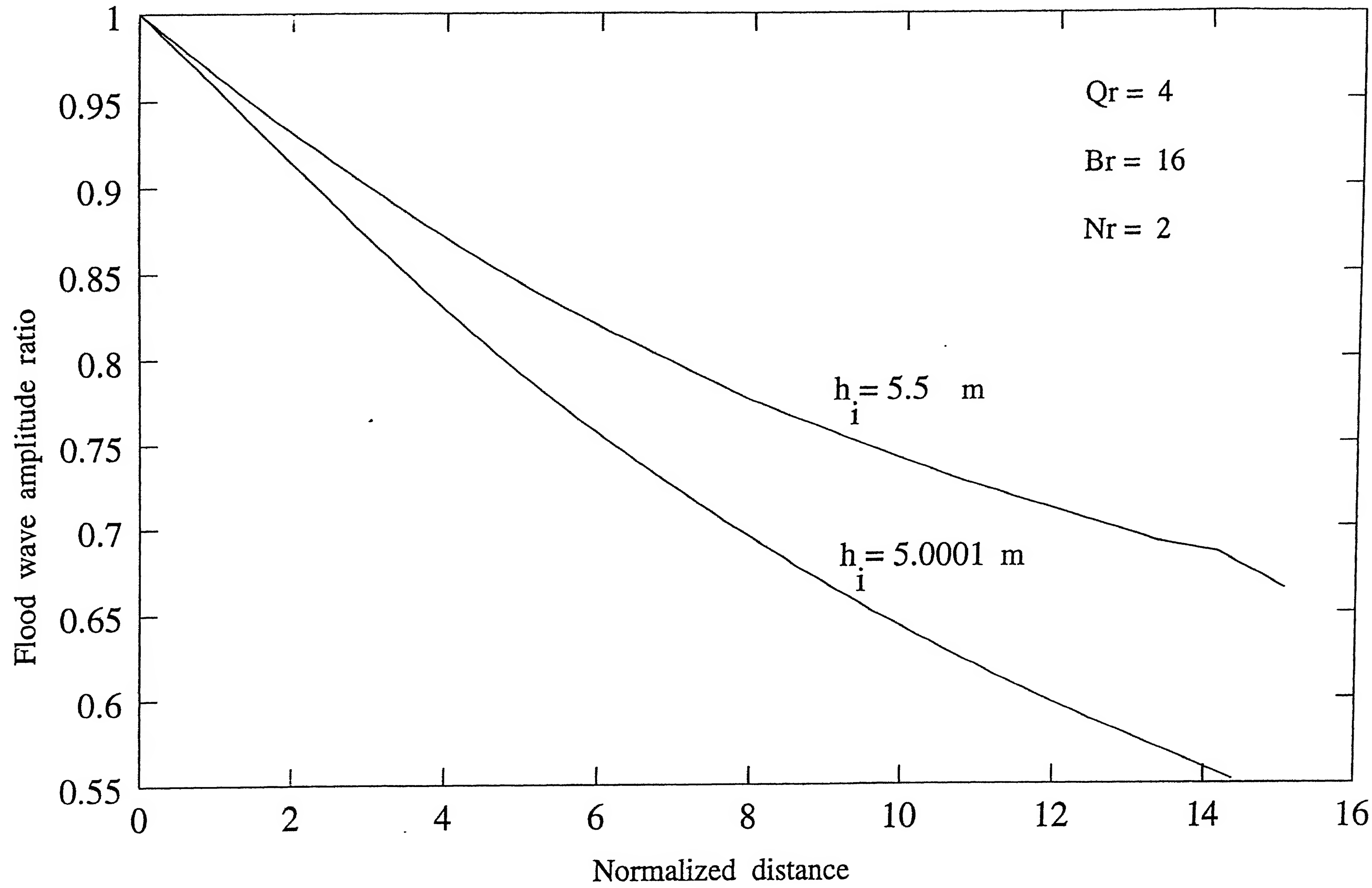


Fig 3.3.2 Effect of initial flow depth on Flood Peak Subsidence

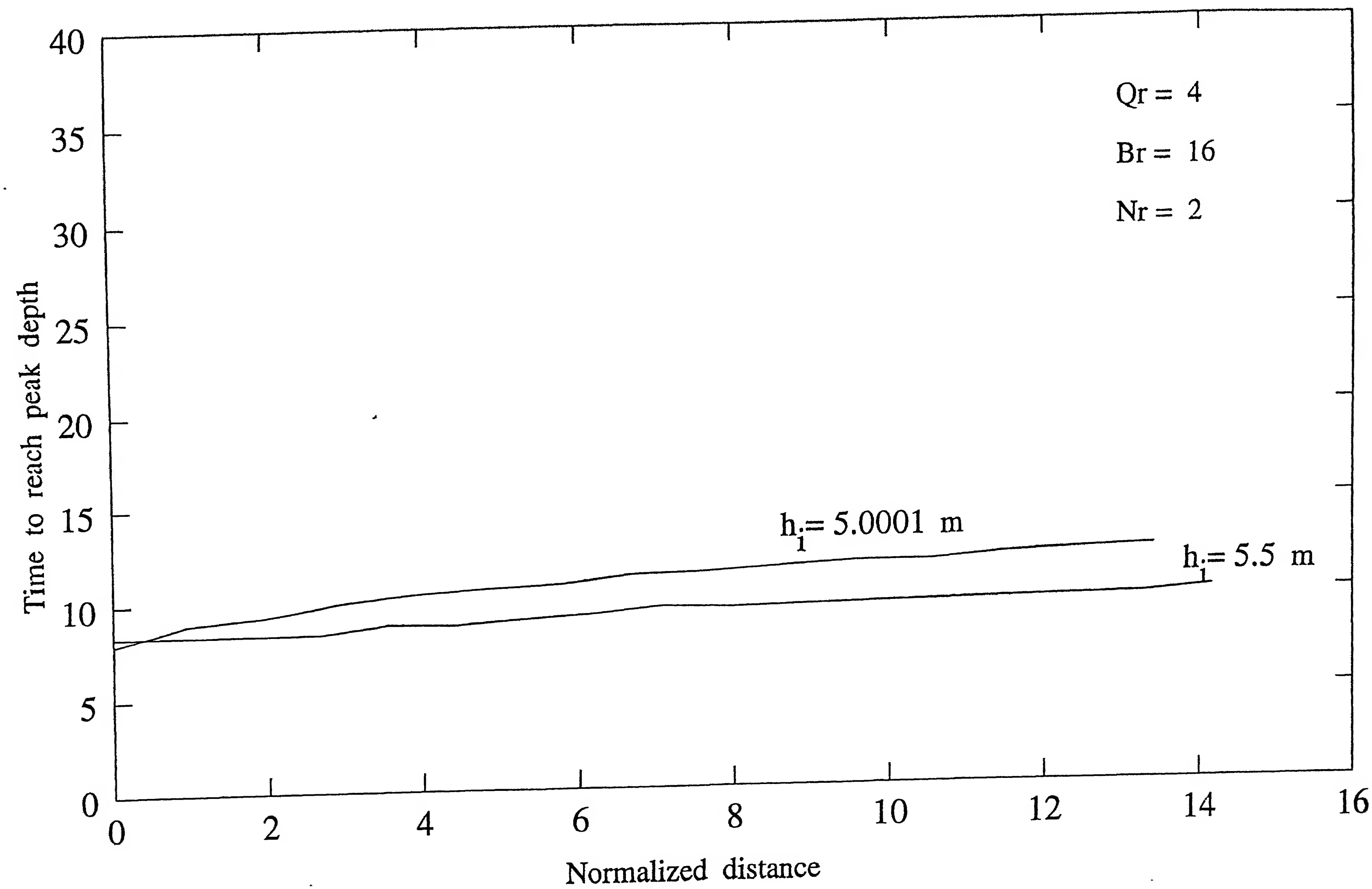


Fig 3.33 Effect of initial flow depth on Time to reach peak depth at various locations

CHAPTER IV

CONCLUSIONS

4.1 CONCLUSIONS

Various conclusions made after the study of the results are as follows -

(i) Results obtained from two - dimensional implicit model for flood routing without considering the inertia terms slightly differ from the one - dimensional model results developed by Mahapatra .K. .

(ii) Almost similar results were obtained with one - dimensional model for peak subsidence with $Q_r = 4$ and $N_r = 2$ and $B_r = 4,16$.

(iii) Flood peak subsidence was observed for all sets of parameters .Time to reach peak depth increases with an increase in B_r values .

(iv) The observed difference between the flood plains and the river cells was not much . Runs were made with initial uniform flow depth up to 5.00001 m and it was observed that as soon as the depth in the flood plain cells becomes 10 cm high ,then the water levels are almost found to be equal in flood plain cells and river cells .

(v) Seeing the variation of different parameters on the flood peak subsidence ,it is concluded that the maximum rate of flood peak subsidence is observed with $B_r = 16$ and $N_r = 2$.

4.2 RECOMMENDATIONS

(i) The same case may be studied by taking the initial flow depth below the flood bank .i.e. the flow is only in the river .

(ii) In the development of the model the physical features of the flood plains should be considered such as roads , dikes etc.

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APPENDIX - A

```
   TWO DIMENSIONAL MATHEMATICLA MODEL FOR FLOOD ROUTING
   L=NUMBER OF CELLS IN LONGITUDINAL DIRECTION
   N=NUMBER OF CELLS IN THE TRANSVERSE DIRECTION
   JN,NP=NUMBER OF CELLS IN THE COMPUTATIONAL DOMAIN
   T=TIME UNDER CONSIDERATION
   DT=INCREMENT IN TIME INTERVAL
   HD=DEPTH OF BED OF UPSTREAM RIVER CELL FROM DATUM
   HM=HEIGHT OF THE WETTED SURFACE IN RIVER
   HB=BEPTH OF FLOOD BANK FROM RIVER BED
   DZCHK=MINIMUM ALLOWABLE CHANGE IN WATER LEVEL
   DZMAX=MAXIMUM CHANGE IN WATER LEVEL
   TLAST=END OF THE COMPUTATIONAL TIME
   TPEAK=TIME TO REACH PEAK DEPTH OF INFLOW HYDROGRAPH
   TGRA=TIME OF CENTRE OF GRAVITY OF INFLOW HYDROGRAPH
   RN=MANNING'S ROUGHNESS COEFFICIENT OF RIVER
   FN=MANNING'S ROUGHNESS COEFFICIENT OF FLOOD PLAIN
   HIR=INITIAL UNIFORM FLOW DEPTH OF RIVER
   HIF=INITIAL UNIFORM FLOW DEPTH OF FLOOD PLAIN
   QD=RATIO OF PEAK DISCHARGE TO BASE FLOW DISCHARGE
   XF1U,XF2U=X-COORDINATES OF FLOOD PLAIN ON UPSTREAM SIDE
   XF1D,XF2D=X-COORDINATES OF FLOOD PLAIN ON DOWNSTREAM  SIDE
   YF1U,YF2U=Y-COORDINATES OF FLOOD PLAIN ON UPSTREAM SIDE
   YF1D,YF2D=Y-COORDINATES OF FLOOD PLAIN ON DOWNSTREAM  SIDE
   XR1U,XR2U=X-COORDINATES OF RIVER ON UPSTREAM SIDE
   XR1D,XR2D-X-COORDINATES OF RIVER ON DOWNSTREAM SIDE
   YR1U,YR2U=Y-COORDINATES OF RIVER ON UPSTREAM SIDE
   YR1D,YR2D=Y-COORDINATES OF RIVER ON DOWNSTREAM SIDE
   H1=DEPTH OF WATER LEVEL FROM BED AT KNOWN TIME
   H2=DEPTH OF WATER LEVEL FROM BED AT NEW TIME
   Z1=DEPTH OF WATER LEVEL FROM DATUM AT KNOWN TIME
   Z2=DEPTH OF WATER LEVEL FROM DATUM AT NEW TIME
   QQ=SOURCE TERM( NET DISCHARGE)
   A1=ELEMENTS OF THE JACOBIAN MATRIX
   BT,BL,BB,BR=LENGTHS OF THE TOP,LEFT,BOTTOM AND RIGHT SIDES OF EACH CELL
   DXT,DXL,DXR,DXB=C/C DISTANCE OF CELLS ON TOP,LEFT,RIGHT AND BOTTOM OF
                ANY PARTICULAR CELL
   CB,CL,CR,CT=ARE THE CORRESPONDING AREA VECTORS TO ANY CELL FROM BOTTOM,
             LEFT,RIGHT AND TOP CELLS
   WRKSP1,WRKSP2,WRKSP3=WORK SPACE PROVIDED FOR NAG ROUTINE
   RESIDS,CHNGS,APARAM,CONRES,CONCHN= GOVERN THE COVERGENCE OF NAG ROUTINE
                                     AS GIVEN IN NAG MANUAL
   A,B,C,D,E=INPUTS TO THE NAG ROUTINE
   X=CHANGE IN THE WATER LEVELS AFTER EVERY ITERATIONS
   IMPLICIT DOUBLE PRECISION(A-H,O-Z)
   EXTERNAL D03EBF
   PARAMETER(NP=209)
   DIMENSION H1(100,11),H2(100,11),Z1(100,11),Z2(100,11),A1(NP,NP),
 1 QQ(100,11),BT(1500),BL(1500),BB(1500),BR(1500),CR(1500),
 1 CB(1500),CL(1500),DXL(1500),DXR(1500),DXT(1500),DXB(1500)
 1 ,WRKSP1(100,11),WRKSP2(100,11),WRKSP3(100,11),RESIDS(250)
 * ,CHNGS(250),CT(1500),AC(1500),
 * A(100,11),B(100,11),C(100,11),D(100,11),E(100,11),X(100,11)
   OPEN (UNIT=1,FILE='IND')
   OPEN (UNIT=2,FILE='OUTDT')
   READ(1,*) L,N,JN,DT,HD,HB,DZCHK
   READ(1,*) TLAST,TPEAK,TGRA,HUI,HUP,HUB
   READ(1,*) RN,FN,SO HIR,HIF,QD,APARAM,CONRES,CONCHN
   READ(1,*) XF1U,XR1U,XR2U,XF2U,XF1D,XR1D,XR2D,XF2D
   READ(1,*) YF1U,YR1U,YR2U,YF2U,YF1D,YR1D,YR2D,YF2D
   CALL AREA(N,XF1U,XR1U,XR2U,XF2U,XF1D,XR1D,XR2D,XF2D,
 1 YF1U,YR1U,YR2U,YF2U,YF1D,YR1D,YR2D,YF2D,
 1 BT,BL,BB,BR,DXT,DXL,DXB,DXR,CT,CL,CB,CR,AC)
   DX=XF1D/N
   BRA=(YF2U-YF1U)/(YR2U-YR1U)
   NRA=FN/RN
   WRITE(2,*) 'NR = ',NRA,'  QF = ', QD,' BR = ',BRA,' DX = ',DX
```

```
      T=0.0
      DO11J=1,L
      DO11I=1,N
      IF(J.EQ.6)THEN
      H1(I,J)=HIR
      Z1(I,J)=H1(I,J)+HD-(I-1)*DX*SO
      ELSE
      H1(I,J)=HIF
      Z1(I,J)=H1(I.J)+HD+HB-(I-1)*DX*SO
      ENDIF
11    CONTINUE
      IF(HIR LT.HB)THEN
      HM=HIR
      ELSE
      HM=HB
      ENDIF
      QS=(BB(1)*HIF)**(5./3.)*SQRT(SO)/(FN*(BB(1)+HIF)**(2./3.))
      QI=BB(N+1)*HIF**(5./3.)*SQRT(SO)/FN
      QN=BB(4.0*N+1)*HIF**(5./3.)*SQRT(SO)/FN
      QF1=(BB(5.0*N+1)*HIR)**(5./3 )*SQRT(SO)
      QF2=(RN*(BB(5.0*N+1)+2.0*HM)**(2 /3.))
      QF=QF1/QF2
5     T=T+DT
      DO12J=1,L
      DO12I=1,N
      H2(I,J)=H1(I,J)
12    CONTINUE
      TDIF=TGRA-TPEAK
      QM=QF+(QD-1)*QF*EXP((TPEAK-T)/TDIF)*(T/TPEAK)**(TPEAK/TDIF)
7     DX=XF1D/N
      DO6J=1,L
      DO6I=1,N
      IF(J.EQ.5)THEN
      HB=0.0
      ELSE
      HB=5.0
      ENDIF
6     Z2(I,J)=H2(I.J)+HD+HB-(I-1)*DX*SO
      DO13J=1,L
      DO13I=1.N-1
      IE=(J-1)*N+I
      IB=IE+1-J
      QC=AC(IE)*(H2(I,J)-H1(I,J))/DT
      CALL TOPBOTM(I,J.IE,FN,RN.Z1,Z2,H2,HB,BT
     1 ,BB,CT,CB,DXT,DXB,QS,QI,QN,QM,QT,QB)
      IF(J.EQ.1)THEN
      CALL LTRT(I,J,J+1,IE,FN,Z1,Z2,H2,BR,CR,DXR,Q)
      F=QC-QT-QB-Q
      ENDIF
      IF(J.GT 1.AND.J.LT 5 OR.J.GT 7 AND.J.LT.11)THEN
      CALL LTRT(I,J,J+1,IE,FN,Z1,Z2,H2,BR,CR,DXR,QR)
      CALL LTRT(I,J.J-1,IE.FN,Z1,Z2.H2,BL,CL,DXL,QL)
      F=QC-QT-QB-QR-QL
      ENDIF
      IF(J EQ.5)THEN
      CALL LTRT(I,J,J-1,IE,FN,Z1,Z2,H2,BL,CL,DXL,QL)
      CALL WEIRLINK(I,J+1,I,IE,BR,CR.H1,H2,HB,QR)
      F=QC-QT-QB+QR-QL
      ENDIF
      IF(J.EQ.6)THEN
      CALL WEIRLINK(I,J,J-1,IE,BL,CL,H1,H2,HB,QL)
      CALL WEIRLINK(I,J,J+1,IE,BR,CR,H1,H2,HB,QR)
      F=QC-QT-QB-QL-QR
      ENDIF
      IF(J.EQ 7)THEN
      CALL LTRT(I,J,J+1,IE,FN,Z1,Z2,H2,BR,CR,DXR,QR)
```

```
      CALL WEIRLINK(I,J-1,J,IE,BL,CL,H1,H2,HB,QL)
      F=QC-QT-QB-QR+QL
      ENDIF
      IF(J.EQ.11)THEN
      CALL LTRT(I,J,J-1,IE,FN,Z1,Z2,H2,BL,CL,DXL,Q)
      F=QC-QT-QB-Q
      ENDIF
      QQ(I,J)=F
13    CONTINUE
      CALL JACOB(N,JN,FN,RN,Z1,Z2,H1,H2,DT,AC,DXT,DXL,DXB,DXR,
     1 BT,BL,BB,BR,CT,CL,CB,CR,HB,A1)
      DZMAX=0.0
      DO J=1,L
      DO I=1,N-1
      J1=(J-1)*(N-1)+I
      C(I,J)=A1(J1,J1)
      IF(J.GT.1)THEN
      A(I,J)=A1(J1,J1-N+1)
      ENDIF
      IF(I.GT.1)THEN
      B(I,J)=A1(J1,J1-1)
      ENDIF
      IF(I.LT.N-1)THEN
      D(I,J)=A1(J1,J1+1)
      ENDIF
      IF(J.LT.11)THEN
      E(I,J)=A1(J1,J1+N-1)
      ENDIF
      ENDDO
      ENDDO
      DO I=1,N-1
      DO J=1,L
      X(I,J)=0.0
      ENDDO
      ENDDO
      CALL D03EBF(19,11,100,A,B,C,D,E,QQ,X,APARAM,250,0,ITUSED,1,1,1
     * ,CONRES,CONCHN,RESIDS,CHNGS,WRKSP1,WRKSP2,WRKSP3,0)
      DO15I=1,N-1
      DO15J=1,L
      IF(ABS(X(I,J)).LT.DZMAX)THEN
      DZMAX=DZMAX
      ELSE
      DZMAX=ABS(X(I,J))
      ENDIF
15    CONTINUE
      DO16J=1,L
      DO16I=1,N-1
16    H2(I,J)=H2(I,J)-X(I,J)
      DO17J=1,L
17    H2(N,J)=H2(N-1,J)
      WRITE(*,*) 'DZMAX=',DZMAX,' T=',T, '   ITUSED =',ITUSED
      IF(DZMAX.GT.DZCHK)GOTO7
      WRITE(2,*) "T=",T
      WRITE(2,19) ((H2(I,J),J=1,L),I=1,N)
19    FORMAT(5F7.5,2X,6F7.5)
      DO18J=1,L
      DO18I=1,N
18    H1(I,J)=H2(I,J)
      DO14J=1,L
      DO14I=1,N
      IF(J.EQ.6)THEN
      Z1(I,J)=H1(I,J)+HD-(I-1)*DX*50
      ELSE
      Z1(I,J)=H1(I,J)+HD+HB-(I-1)*DX*50
      ENDIF
14    CONTINUE
```

```
      IF(T.LT.TLAST)GOTO5
      STOP
      END
C     ELEMENTS OF CONNECTIVITY MATRIX CONNECTING
C     GLOBAL NODES TO LOCAL NODES.
      SUBROUTINE CONNECT(N,C)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION C(1500,1500)
      DO3J=1,4
      IF(J.EQ.1)THEN
      DO4I=1,N
4     C(I,J)=I
      DO5I=N+1,2*N
5     C(I,J)=I+1
      DO6I=2*N+1,3*N
6     C(I,J)=I+2
      DO7I=3*N+1,4*N
7     C(I,J)=I+3
      DO8I=4*N+1,5*N
8     C(I,J)=I+4
      DO9I=5*N+1,6*N
9     C(I,J)=I+5
      DO18I=6*N+1,7*N
18    C(I,J)=I+6
      DO22I=7*N+1,8*N
22    C(I,J)=I+7
      DO23I=8*N+1,9*N
23    C(I,J)=I+8
      DO24I=9*N+1,10*N
24    C(I,J)=I+9
      DO25I=10*N+1,11*N
25    C(I,J)=I+10
      ENDIF
      IF(J.EQ.2)THEN
      DO19I=1,11*N
19    C(I,J)=C(I,J-1)+1
      ENDIF
      IF(J.EQ.3)THEN
      DO20I=1,11*N
20    C(I,J)=C(I,J-2)+N+2
      ENDIF
      IF(J.EQ.4)THEN
      DO21I=1,11*N
21    C(I,J)=C(I,J-1)-1
      ENDIF
3     CONTINUE
      RETURN
      END
C     SUBROUTINE TO CALCULATE THE AREA OF
C     EACH CELL AND GIVE THE CORRESPONDING AREA VECTORS.
      SUBROUTINE AREA(N,XF1U,XR1U,XR2U,XF2U,XF1D,XR1D,XR2D,
     1 XF2D,YF1U,YR1U,YR2U,YF2U,YF1D,YR1D,YR2D,YF2D,BT,BL,BB,
     1 BR,DXT,DXL,DXB,DXR,CT,CL,CB,CR,AC)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION X(1500),Y(1500),EX(50),EY(50),C(1500,1500),AC(1500)
     1 ,BT(1500),BL(1500),BB(1500),BR(1500),CT(1500),CL(1500),CB(1500),
     1 CR(1500),DXT(1500),DXL(1500),DXB(1500),DXR(1500)
      NEL=N*11
      CALL CONNECT(N,C)
      DO11I=1,N+1
      X(I)=XF1U+(I-1)*(XF1D-XF1U)/N
      DO11J=1,11
11    X(J*(N+1)+I)=X(I)
      DO12I=1,N+1
      DO12J=1,6
      YU=YF1U+(YR1U-YF1U)*(J-1)/5.
```

```
      YD=YF1D+(YR1D-YF1D)*(J-1)/5.
      Y((J-1)*(N+1)+I)=YU-(I-1)*(YU-YD)/N
      YU=YR2U+(YF2U-YR2U)*(J-1)/5.
      YD=YR2D+(YF2D-YR2D)*(J-1)/5
12    Y((J+5)*(N+1)+I)=YU-(I-1)*(YU-YD)/N
      DO20IE=1,NEL
      DXT(IE)=1.0
      DXL(IE)=1.0
      DXB(IE)=1.0
      DXR(IE)=1.0
      CT(IE)=1.0
      CL(IE)=1.0
      CB(IE)=1.0
20    CR(IE)=1.0
      DO21IE=1,NEL
      DO22I=1,4
      J=C(IE,I)
      EX(I)=X(J)
22    EY(I)=Y(J)
      BT(IE)=EY(4)-EY(1)
      BB(IE)=EY(3)-EY(2)
      B3=EX(3)-EX(4)
      B4=EY(1)-EY(2)
      B5=EY(3)-EY(4)
      B6=B4+B5
      AC1=BT(IE)+B3
      AC2=1.0/2.0*B3*B6
      AC(IE)=AC1+AC2
      BL(IE)=SQRT(B3**2.0+B4**2.0)
      BR(IE)=SQRT(B3**2.0+B5**2.0)
C     IF(IE.EQ.1.OR.IE.EQ.N+1.OR.IE.EQ.2*N+1.OR.IE.EQ.3*N+1.OR.
C    1 IE.EQ.4*N+1.OR.IE.EQ.5*N+1.OR.IE.EQ.6*N+1.OR.IE.EQ.N.OR.IE.EQ.
C    1  2*N.OR.IE.EQ.3*N.OR.IE.EQ.4*N.OR.IE.EQ.5*N.OR.IE.EQ.6*N.OR.
C    1  IE.EQ.7*N)GOTO21
      YT=(EY(4)+EY(1))/2.0
      YB=(EY(3)+EY(2))/2.0
      YC=(YT+YB)/2.0
      XC=(EX(2)+EX(1))/2.0
      IF(IE.EQ.1.OR.IE.EQ.N+1.OR.IE.EQ.2*N+1.OR.IE.EQ.3*N+1.OR.
     1 IE.EQ.4*N+1.OR.IE.EQ.5*N+1.OR.IE.EQ.6*N+1.OR.IE.EQ.10*N+1.OR.
     1 IE.EQ.7*N+1.OR.IE.EQ.8*N+1.OR.IE.EQ.9*N+1)GOTO5
      J=C(IE-1,1)
      EX1=X(J)
      EY1=Y(J)
      J=C(IE-1,4)
      EX8=X(J)
      EY8=Y(J)
      YT1=(EY8+EY1)/2.0
      YC1=(YT1+YT)/2.0
      XC1=(EX(1)+EX1)/2.0
      DH=SQRT((YC1-YC)**2.0+(XC1-XC)**2.0)
      CT(IE)=(XC-XC1)/DH
      DXT(IE)=DH
5     IF(IE.EQ.N.OR.IE.EQ.2*N.OR.IE.EQ.3*N.OR.IE.EQ.4*N.OR.IE.EQ.5*N
     1 .OR.IE.EQ.6*N.OR.IE.EQ.7*N.OR.IE.EQ.8*N.OR.IE.EQ.9*N.OR.
     1 IE.EQ.10*N.OR.IE.EQ.11*N)GOTO6
      J=C(IE+1,2)
      EX4=X(J)
      EY4=Y(J)
      J=C(IE+1,3)
      EX5=X(J)
      EY5=Y(J)
      YB1=(EY4+EY5)/2.0
      YC1=(YB+YB1)/2.0
      XC1=(EX(2)+EX4)/2.0
      DH=SQRT((XC1-XC)**2.0+(YC1-YC)**2.0)
```

```
      CB(IE)=(XC1-XC)/DH
      DXB(IE)=DH
6     IF(IE.GT.N)THEN
      J=C(IE-N,1)
      EX2=X(J)
      EY2=Y(J)
      J=C(IE-N,2)
      EX3=X(J)
      EY3=Y(J)
      YT1=(EY2+EY(1))/2.0
      YB1=(EY3+EY(2))/2.0
      YC1=(YT1+YB1)/2.0
      DXL(IE)=(YC-YC1)
      CL(IE)=B3/BL(IE)
      ENDIF
      IF(IE.LT.10*N)THEN
      J=C(IE+N,3)
      EX6=X(J)
      EY6=Y(J)
      J=C(IE+N,4)
      EX7=X(J)
      EY7=Y(J)
      YT1=(EY7+EY(4))/2.0
      YB1=(EY6+EY(3))/2.0
      YC1=(YT1+YB1)/2.0
      DXR(IE)=YC1-YC
      CR(IE)=B3/BR(IE)
      ENDIF
21    CONTINUE
      RETURN
      END
C     SUBROUTINE TO CALCULATE THE DISCHARGE BETWEEN THE
C     NEXT TO RIVER CELLS AND THE RIVER CELLS BY USING
C     THE WEIR FORMULA
      SUBROUTINE WEIRLINK(I,K,J,IE,DX,CO,H1,H2,HB,Q)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION DX(1500),CO(1500),H1(100,11),H2(100,11)
      C1=2./3.*SQRT(2.0*9.81)*DX(IE)
      C2=C1*1.5
      IF(H2(I,K).LT.HB)THEN
      W=0.0
      H=H2(I,J)
      CDO=0.0
      CDW=1.06
      D=1.0
      ELSE
      IF((H2(I,K)-HB).LE.H2(I,J))THEN
      W=H2(I,K)-HB
      H=H2(I,J)-W
      D=1.0
      ELSE
      W=H2(I,J)
      H=H2(I,K)-HB-W
      D=-1.0
      ENDIF
      IF(W.LT.(0.667*(W+H)))THEN
      CDO=0.0
      CDW=0.611+0.075*(H/W)
      ELSE
      CDO=0.611-0.175*(W/(H+W))
      CDW=0.0
      ENDIF
      ENDIF
      IF(H.LE.0.0)THEN
      Q=0.0
      ELSE
```

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      Q=D*(C1*CDU*H**1.5+C2*CDO*W*H**0.5)*CO(IE)
      Q1=C1*CDU*H**1.5
      Q2=C2*CDO*W*H**0.5
      Q=D*(Q1+Q2)*CO(IE)
      ENDIF
      RETURN
      END
      SUBROUTINE CALCULATES THE DISCHARGE COMING
      IN OR GOING OUT FROM/TO TOP AND BOTTOM CELLS
      SUBROUTINE TOPBOTM(I,J,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,
     1 CT,CB,DXT,DXB,QS,QI,QN,QM,QT,QB)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION Z1(100,11),Z2(100,11),H2(100,11),BT(1500),BB(1500)
     1 ,CT(1500),CB(1500),DXT(1500),DXB(1500)
      IF(I.EQ.1)GOTO010
      HT=(H2(I,J)+H2(I-1,J))/2.0
      IF(Z2(I,J).LT.Z2(I-1,J))THEN
      ST=SQRT((Z2(I-1,J)-Z2(I,J))/DXT(IE))
      ELSE
      ST=-SQRT((Z2(I,J)-Z2(I-1,J))/DXT(IE))
      ENDIF
0     HBT=(H2(I,J)+H2(I+1,J))/2.0
      IF(Z2(I,J).LT.Z2(I+1,J))THEN
      SB=SQRT((Z2(I+1,J)-Z2(I,J))/DXB(IE))
      ELSE
      SB=-SQRT((Z2(I,J)-Z2(I+1,J))/DXB(IE))
      ENDIF
      IF(J.EQ.1.OR.J.EQ.11)THEN
      IF(I.EQ.1)THEN
      QT=QS
      ELSE
      QT=BT(IE)**(5./3.)*CT(IE)*HT**(5./3.)*ST
     1 /(FN*(BT(IE)+HT)**(2./3.))
      ENDIF
      QB=BB(IE)**(5./3.)*CB(IE)*HBT**(5./3.)*SB
     1 /(FN*(BB(IE)+HBT)**(2./3.))
      ENDIF
      IF(J.GE.2.AND.J.LE.5.OR.J.GE.7.AND.J.LE.10)THEN
      IF(I.EQ.1)THEN
      IF(J.GE.2.AND.J.LE.4.OR.J.GE.8.AND.J.LE.10)QT=QI
      IF(J.EQ.5.OR.J.EQ.7)QT=QN
      ELSE
      QT=BT(IE)*HT**(5./3.)*ST*CT(IE)/FN
      ENDIF
      QB=BB(IE)*HBT**(5./3.)*SB*CB(IE)/FN
      ENDIF
      IF(J.EQ.6)THEN
      IF(I.EQ.1)THEN
      QT=QM
      ELSE
      IF(HT.LT.HB)THEN
      PT=BT(IE)+2.0*HT
      ELSE
      PT=BT(IE)+2.0*HB
      ENDIF
      QT=(BT(IE)*HT)**(5./3.)*ST*CT(IE)/(RN*PT**(2./3.))
      ENDIF
      IF(HBT.LT.HB)THEN
      PB=BB(IE)+2.0*HBT
      ELSE
      PB=BB(IE)+2.0*HB
      ENDIF
      QB=(BB(IE)*HBT)**(5./3.)*SB*CB(IE)/(RN*PB**(2./3.))
      ENDIF
      RETURN
      END
```

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      SUBROUTINE TO CALCULATE THE DISCHARGE COMING IN
      OR GOING OUT FROM/TO LEFT AND RIGHT CELLS
      SUBROUTINE LTRT(I,J,K,IE,FN,Z1,Z2,H2,BO,CO,DX,Q)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION Z1(100,11),Z2(100,11),H2(100,11),BO(1500)
*     ,CO(1500),DX(1500)
      H=(H2(I,J)+H2(I,K))/2.0
      IF(Z2(I,J).LT.Z2(I,K))THEN
      SL=SQRT((Z2(I,K)-Z2(I,J))/DX(IE))
      ELSE
      SL=-SQRT((Z2(I,J)-Z2(I,K))/DX(IE))
      ENDIF
      Q=BO(IE)*H**(5./3.)*SL*CO(IE)/FN
      RETURN
      END
      SUBROUTINE TO CALCULATE THE DIFFERENTIAL TERMS COTRIBUTING
      TO MAIN CELL FROM LEFT/RIGHT THROUGH WEIR LINK .
      SUBROUTINE DIFFWEIRLINK(I,K,J,IE,DX,CO,H1,H2,HB,DQR,DQF)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION DX(1500),H1(100,11),H2(100,11),CO(1500)
      C1=2./3.*SQRT(2.0*9.81)*DX(IE)
      C2=C1*1.5
      IF(H2(I,K).LT.HB)THEN
      H=H2(I,J)
      CDW=1.06
      DQR=0.0
      DQF=C1*CDW*1.5*H**0.5*CO(IE)
      ELSE
      IF((H2(I,K)-HB).LE.H2(I,J))THEN
      W=H2(I,K)-HB
      H=H2(I,J)-H2(I,K)+HB
      IF(H.LE.0.0)THEN
      DQR=0.0
      DQF=0.0
      ELSE
      A1=-C1*0.611*1.5*(H**0.5)
      A2=-C1*0.075*(W*2.5*(H**1.5)+(H**2.5))/(W**2.0)
      A3=C2*0.611*(H**0.5-W/(2.0*SQRT(H)))
      A4=-C2*0.175*W*((H**0.5)*2.0-W*0.5/SQRT(H))/(W+H)
      A5=C1*0.075*2.5*H**1.5/W
      A6=C2*0.611*W*0.5/SQRT(H)
      A7=-C2*0.175*(W**2.0)*((W+H)*0.5/SQRT(H)-SQRT(H))/((W+H)**2.0)
      IF(W.LT.(0.66667*(W+H)))THEN
      DQR=(A1+A2)*CO(IE)
      DQF=(-A1+A5)*CO(IE)
      ELSE
      DQR=(A3+A4)*CO(IE)
      DQF=(A6+A7)*CO(IE)
      ENDIF
      ENDIF
      ELSE
      W=H2(I,J)
      H=H2(I,K)-HB-H2(I,J)
      IF(H.LE.0.0)THEN
      DQR=0.0
      DQF=0.0
      ELSE
      A1=C1*0.611*1.5*H**0.5
      A2=C1*0.075*2.5*(H**1.5)/W
      A3=C2*0.611*W*0.5/SQRT(H)
      A4=-C2*0.175*(W**2.0)*((W+H)*0.5/SQRT(H)-SQRT(H))/((W+H)**2.0)
      A5=-C1*0.075*(W*2.5*(H**1.5)+(H**2.5))/(W**2.0)
      A6=C2*0.611*(H**0.5-W*0.5/SQRT(H))
      A7=-C2*0.175*W*(2.0*SQRT(H)-W*0.5/SQRT(H))/(W+H)
      IF(W.LT.(0.66667*(W+H)))THEN
      DQR=(-A1-A2)*CO(IE)
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      DQF=(A1-A5)*CO(IE)
      ELSE
      DQR=(-A3-A4)*CO(IE)
      DQF=(-A6-A7)*CO(IE)
      ENDIF
      ENDIF
      ENDIF
      ENDIF
      RETURN
      END
      SUBROUTINE TO CALCULATE THE DIFFERENTIAL TERMS OF DISCHARGE
      CONTRIBUTING TO THE MAIN CELLS FROM LEFT AND RIGHT CELLS .
      SUBROUTINE DIFFLTRT(I,J,K,IE,FN,Z1,Z2,H2,BO,CO,DX,DQ1,DQ2)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION Z1(100,11),Z2(100,11),H2(100,11),BO(1500)
     * ,CO(1500),DX(1500)
      H=H2(I,J)+H2(I,K)
      IF(Z2(I,J).LT Z2(I,K))THEN
      SL=SQRT(Z2(I,K)-Z2(I,J))
      DSL1=-1.0/(2 0*SL)
      DSL2=1.0/(2.0*SL)
      ELSE
      SL=-SQRT(Z2(I,J)-Z2(I,K))
      DSL1=1.0/(2.0*SL)
      DSL2=-1.0/(2.0*SL)
      ENDIF
      A1=BO(IE)/(2.0**(5./3.)*FN*SQRT(DX(IE)))
      IF(ABS(SL).LE.0.0)THEN
      DQ1=0.0
      DQ2=0.0
      ELSE
      DQ1=A1*(SL*(5./3.)*H**(2./3.)+H**(5./3.)*DSL1)
      DQ2=A1*(SL*(5./3. )*H**(2./3.)+H**(5./3.)*DSL2)
      ENDIF
      RETURN
      END
      SUBROUTINE CALCULATES THE DIFFERENTIAL ELEMENTS OF
      TOP AND BOTTOM CELLS CONTRIBUTING THE MAIN CELL .
      SUBROUTINE DIFFTOPBOTM(I,J,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,
     1 DXT,DXB,DQT1,DQT2,DQB1,DQB2)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION Z1(100,11),Z2(100,11),H2(100,11)
     * ,BT(1500),CT(1500),BB(1500),
     1 CB(1500),DXT(1500),DXB(1500)
      IF(I.EQ.1)GOTO5
      HT=H2(I,J)+H2(I-1,J)
      IF(Z2(I,J) LT.Z2(I-1,J))THEN
      ST=SQRT(Z2(I-1,J)-Z2(I,J))
      DST1=-1 0/(2.0*ST)
      DST2=1 0/(2.0*ST)
      ELSE
      ST=-SQRT(Z2(I,J)-Z2(I-1,J))
      DST1=1.0/(2.0*ST)
      DST2=-1 0/(2.0*ST)
      ENDIF
      HBT=H2(I,J)+H2(I+1,J)
      IF(Z2(I,J).LT.Z2(I+1,J))THEN
      SB=SQRT(Z2(I+1,J)-Z2(I,J))
      DSB1=-1.0/(2 0*SB)
      DSB2=1.0/(2.0*SB)
      ELSE
      SB=-SQRT(Z2(I,J)-Z2(I+1,J))
      DSB1=1.0/(2.0*SB)
      DSB2=-1 0/(2.0*SB)
      ENDIF
      IF(J EQ 1.OR J EQ.11)THEN
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IF(I.EQ 1)THEN
DQT1=0.0
DQT2=0.0
ELSE
A1=BT(IE)**(5./3 )*CT(IE)/(FN*2.0*SQRT(DXT(IE)))
A3=2 0*BT(IE)+HT
A5=(ST*(5 /3 )*HT**(2./3 )+HT**(5./3.)*DST1)/A3**(2./3.)
A6=HT**(5 /3 )*ST*2.0/(3 0*A3**(5./3.))
A9=(ST*(5 /3 )*HT**(2 /3.)+HT**(5./3.)*DST2)/A3**(2./3.)
DQT1=A1*(A5-A6)
DQT2=A1*(A9-A6)
ENDIF
A2=BB(IE)**(5./3 )*CB(IE)/(FN*2.0*SQRT(DXB(IE)))
A4=2.0*BB(IE)+HBT
A7=(SB*(5./3 )*HBT**(2./3.)+HBT**(5./3.)*DSB1)/A4**(2./3.)
A8=HBT**(5./3.)*SB*2 0/(3.0*A4**(5./3.))
A10=(SB*(5./3 )*HBT**(2./3.)+HBT**(5./3.)*DSB2)/A4**(2./3.)
DQB1=A2*(A7-A8)
DQB2=A2*(A10-A8)
ENDIF
IF(J.GE.2.AND.J.LE.5 OR.J.GE.7.AND.J.LE.10)THEN
A1=BT(IE)*CT(IE)/(FN*2 0)*(5./3.)*SQRT(DXT(IE)))
A2=BB(IE)*CB(IE)/(FN*2 0)*(5./3.)*SQRT(DXB(IE)))
IF(I.EQ 1)THEN
DQT1=0.0
DQT2=0.0
ELSE
DQT1=A1*(ST*(5./3.)*HT**(2./3.)+HT**(5./3.)*DST1)
DQT2=A1*(ST*(5./3.)*HT**(2./3.)+HT**(5./3.)*DST2)
ENDIF
DQB1=A2*(SB*(5 /3.)*HBT**(2./3.)+HBT**(5./3.)*DSB1)
DQB2=A2*(SB*(5./3.)*HBT**(2./3.)+HBT**(5./3.)*DSB2)
ENDIF
IF(J.EQ.6)THEN
IF(I.EQ.1)THEN
DQT1=0.0
DQT2=0.0
ELSE
A1=BT(IE)**(5./3.)*CT(IE)/(RN*2.0)*(5./3.)*SQRT(DXT(IE)))
A3=ST*(5./3.)*HT**(2./3.)+HT**(5./3.)*DST1
A4=ST*(5./3.)*HT**(2 /3.)+HT**(5./3.)*DST2
IF((HT/2 0).LT.HB)THEN
PT=BT(IE)+HT
A5=(HT**(5./3 )*ST*(2./3 ))/PT**(5./3.)
A6=A3/PT**(2./3.)
A7=A4/PT**(2./3.)
DQT1=A1*(A6-A5)
DQT2=A1*(A7-A5)
ELSE
PT=BT(IE)+2.0*HB
A5=A3/PT**(2./3.)
A6=A4/PT**(2./3.)
DQT1=A1*A5
DQT2=A1*A6
ENDIF
ENDIF
A2=BB(IE)**(5./3 )*CB(IE)/(RN*2 0)*(5./3.)*SQRT(DXB(IE)))
A8=SB*(5 /3.)*HBT**(2./3 )+HBT**(5./3.)*DSB1
A9=SB*(5 /3.)*HBT**(2./3 )+HBT**(5./3.)*DSB2
IF((HBT/2.0) LT.HB)THEN
PB=BB(IE)+HBT
A5=HBT**(5 /3.)*SB*(2./3.)/PB**(5./3.)
A6=A8/PB**(2./3.)
A7=A9/PB**(2./3.)
DQB1=A2*(A6-A5)
DQB2=A2*(A7-A5)
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      ELSE
      PB=BB(IE)+2.0*HB
      A5=A8/PB**(2./3.)
      A6=A9/PB**(2./3.)
      DQB1=A2*A5
      DQB2=A2*A6
      ENDIF
      ENDIF
      RETURN
      END
      ELEMENTS OF THE JACOBIAN MATRIX
      SUBROUTINE JACOB(N,JN,FN,RN,Z1,Z2,H1,H2,DT,AC,DXT,DXL,DXB,
     1 DXR,BT,BL,BB,BR,CT,CL,CB,CR,HB,DF)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION Z1(100,11),H1(100,11),H2(100,11),AC(1500),DXT(1500),
     1 DXL(1500),DXB(1500),DXR(1500),BT(1500),BL(1500),BB(1500),
     1 CT(1500),CL(1500),CB(1500),CR(1500),DF(JN,JN),Z2(100,11)
     * ,BR(1500)
      DO25I=1,N-1
      IE=I
      CALL DIFFTOPBOTM(IE,1,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFLTRT(IE,1,2,IE,FN,Z1,Z2,H2,BR,CR,DXR,DQ1,DQ2)
      DO25J=1,JN
      IF(J.EQ.I.OR.J.EQ.I+N-1.OR.(I.LT.N-1.AND.J.EQ.I+1).OR.
     1 (I.GT.1.AND.J.EQ.I-1))THEN
      IF(J.EQ.I)DF(I,J)=AC(IE)/DT-DQB1-DQT1-DQ1
      IF(J.EQ.I+1)DF(I,J)=-DQB2
      IF(J.EQ.I-1)DF(I,J)=-DQT2
      IF(J.EQ.I+N-1)DF(I,J)=-DQ2
      ELSE
      DF(I,J)=0.0
      ENDIF
      CONTINUE
      DO J=2,4
      DO26I=J*(N-1)-N+2,J*(N-1)
      IE=I+J-1
      I1=I-(J-1)*(N-1)
      CALL DIFFTOPBOTM(I1,J,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFLTRT(I1,J,J+1,IE,FN,Z1,Z2,H2,BR,CR,DXR,DQR1,DQR2)
      CALL DIFFLTRT(I1,J,J-1,IE,FN,Z1,Z2,H2,BL,CL,DXL,DQL1,DQL2)
      DO26J1=1,JN
      IF(J1.EQ.I.OR.J1.EQ.I+N-1.OR.J1.EQ.I-N+1.OR.(I.LT.J*(N-1).AND.
     1 J1.EQ.I+1).OR.(I.GT.J*(N-1)-N+2.AND.J1.EQ.I-1))THEN
      IF(J1.EQ.I)DF(I,J1)=AC(IE)/DT-DQB1-DQT1-DQL1-DQR1
      IF(J1.EQ.I+1)DF(I,J1)=-DQB2
      IF(J1.EQ.I-1)DF(I,J1)=-DQT2
      IF(J1.EQ.I+N-1)DF(I,J1)=-DQR2
      IF(J1.EQ.I-N+1)DF(I,J1)=-DQL2
      ELSE
      DF(I,J1)=0.0
      ENDIF
6     CONTINUE
      ENDDO
      DO27I=4*N-3,5*N-5
      I1=I-4*N+4
      IE=I+4
      CALL DIFFTOPBOTM(I1,5,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFLTRT(I1,5,4,IE,FN,Z1,Z2,H2,BL,CL,DXL,DQL1,DQL2)
      CALL DIFFWEIRLINK(I1,6,5,IE,BR,CR,H1,H2,HB,DQ1,DQ2)
      DO27J=1,JN
      IF(J.EQ.I.OR.J.EQ.I-N+1.OR.J.EQ.I+N-1.OR.(I.GT.4*N-3.AND.
     1 J.EQ.I-1).OR.(I.LT.5*N-5.AND.J.EQ.I+1))THEN
      IF(J.EQ.I)DF(I,J)=AC(IE)/DT-DQB1-DQT1-DQL1+DQ2
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      IF(J.EQ.I+1)DF(I,J)=-DQB2
      IF(J.EQ.I-1)DF(I,J)=-DQT2
      IF(J.EQ.I+N-1)DF(I,J)=DQ1
      IF(J.EQ.I-N+1)DF(I,J)=-DQL2
      ELSE
      DF(I,J)=0.0
      ENDIF
27    CONTINUE
      DO28I=5*N-4,5*N-5
      I1=I-5*N+5
      IE=I+5
      CALL DIFFTOPBOTM(I1,6,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFWEIRLINK(I1,6,5,IE,BL,CL,H1,H2,HB,DQL1,DQL2)
      CALL DIFFWEIRLINK(I1,6,7,IE,BR,CR,H1,H2,HB,DQR1,DQR2)
      DO28J=1,JN
      IF(J.EQ.I.OR.J.EQ.I-N+1.OR.J.EQ.I+N-1.OR.(I.GT.5*N-4.AND.
     1 J.EQ.I-1).OR.(I.LT.6*N-6.AND.J.EQ.I+1))THEN
      IF(J.EQ.I)DF(I,J)=AC(IE)/DT-DQB1-DQT1-DQL1-DQR1
      IF(J.EQ.I+1)DF(I,J)=-DQB2
      IF(J.EQ.I-1)DF(I,J)=-DQT2
      IF(J.EQ.I+N-1)DF(I,J)=-DQR2
      IF(J.EQ.I-N+1)DF(I,J)=-DQL2
      ELSE
      DF(I,J)=0.0
      ENDIF
28    CONTINUE
      DO22I=6*N-5,7*N-7
      I1=I-6*N+6
      IE=I+6
      CALL DIFFTOPBOTM(I1,7,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFLTRT(I1,7,8,IE,FN,Z1,Z2,H2,BR,CR,DXR,DQR1,DQR2)
      CALL DIFFWEIRLINK(I1,6,7,IE,BL,CL,H1,H2,HB,DQ1,DQ2)
      DO22J=1,JN
      IF(J.EQ.I.OR.J.EQ.I-N+1.OR.J.EQ.I+N-1.OR.(I.GT.6*N-5.AND.
     1 J.EQ.I-1).OR.(I.LT.7*N-7.AND.J.EQ.I+1))THEN
      IF(J.EQ.I)DF(I,J)=AC(IE)/DT-DQB1-DQT1-DQR1+DQ2
      IF(J.EQ.I+1)DF(I,J)=-DQB2
      IF(J.EQ.I-1)DF(I,J)=-DQT2
      IF(J.EQ.I+N-1)DF(I,J)=-DQR2
      IF(J.EQ.I-N+1)DF(I,J)=DQ1
      ELSE
      DF(I,J)=0.0
      ENDIF
22    CONTINUE
      DO J=8,10
      DO23I=J*(N-1)-N+2,J*(N-1)
      IE=I+J-1
      I1=I-(J-1)*(N-1)
      CALL DIFFTOPBOTM(I1,J,IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFLTRT(I1,J,J+1,IE,FN,Z1,Z2,H2,BR,CR,DXR,DQR1,DQR2)
      CALL DIFFLTRT(I1,J,J-1,IE,FN,Z1,Z2,H2,BL,CL,DXL,DQL1,DQL2)
      DO23J1=1,JN
      IF(J1.EQ.I.OR.J1.EQ.I+N-1.OR.J1.EQ.I-N+1.OR.(I.LT.J*(N-1).AND.
     1 J1.EQ.I+1).OR.(I.GT.J*(N-1)-N+2.AND.J1.EQ.I-1))THEN
      IF(J1.EQ.I)DF(I,J1)=AC(IE)/DT-DQB1-DQT1-DQL1-DQR1
      IF(J1.EQ.I+1)DF(I,J1)=-DQB2
      IF(J1.EQ.I-1)DF(I,J1)=-DQT2
      IF(J1.EQ.I+N-1)DF(I,J1)=-DQR2
      IF(J1.EQ.I-N+1)DF(I,J1)=-DQL2
      ELSE
      DF(I,J1)=0.0
      ENDIF
23    CONTINUE
```

```
      ENDDO
      DO24I=10*N-9,11*(N-1)
      I1=I-10*N+10
      IE=I+10
      CALL DIFFTOPBOTM(I1,11.IE,FN,RN,Z1,Z2,H2,HB,BT,BB,CT,CB,DXT,
     1 DXB,DQT1,DQT2,DQB1,DQB2)
      CALL DIFFLTRT(I1,11,10,IE,FN.Z1,Z2,H2,BL,CL,DXL,DQ1,DQ2)
      DO24J=1,JN
      IF(J EQ I OR J EQ.I-N+1 OR (I LT.11*N-11.AND.J.EQ.I+1).OR.
     1 (I GT.10*N-9 AND J EQ.I-1))THEN
      IF(J EQ.I)DF(I,J)=AC(IE)/DI-DQB1-DQT1-DQ1
      IF(J EQ I+1)DF(I,J)=-DQB2
      IF(J.EQ.I-1)DF(I,J)=-DQT2
      IF(J EQ.I-N+1)DF(I,J)=-DQ2
      ELSE
      DF(I.J)=0 0
      ENDIF
24    CONTINUE
      RETURN
      END
```